KSPEU

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
Federal State Budgetary Educational Institution of Higher Education
«KAZAN STATE POWER ENGINEERING UNIVERSITY»
(FSBEI HE «KSPEU»)

APPROVED
Director of the Institute of Digital
Technologies and Economics
_____Zainullin R.R.
«24»_____February_____2026

WORK PROGRAM OF THE DISCIPLINE

B1.M.12 Mathematics

Field of training — 38.03.02 Management

Qualification — Bachelor's Degree

Kazan, 2026

The program was developed by:

Department Name	Position, Academic Degree, Academic Title	Developer's Full Name
Higher Mathematics	Associate Professor, PhD in Phys.-Math. Sciences	Grigoryan T.A.
Higher Mathematics	Associate Professor, PhD in Phys.-Math. Sciences	Fauaz Kh.O.

Approval	Department Name	Date	Protocol No.	Signature
Approved	Higher Mathematics (HM)	11.03.2026	Protocol №4	_____ Head of Department, Dr. of Phys.-Math. Sciences, Assoc. Prof. Sitdikov A. S.
Agreed	Management	10.02.2026	Protocol №5	_____ Head of the Department., Doctor of Social Sciences, prof.Makhiyanova A.V.
Agreed	Educational and Methodological Council of IDTE	24.02.2026	Protocol №6	_____ Director, Ph.D., Associate Professor, Zainullin R.R..
Approved	Scientific Council of IDTE	24.02.2026	Protocol №6	_____ Director, Ph.D., Associate Professor, Zainullin R.R.

1. Goal, Tasks, and Planned Learning Outcomes of the Discipline

The goal of studying the discipline is the theoretical mastery by students of the main sections of linear algebra, mathematical analysis, probability theory and mathematical statistics, necessary for understanding the role of mathematics in professional activities; forming a culture of thinking, the ability to generalize, analyze, perceive information, set goals and choose ways to achieve them; mastering the basic methods of the discipline applied in solving professional tasks and research activities.

Tasks of the discipline:

- to provide an understanding of the role of mathematics in professional activities;
- to introduce the necessary conceptual framework of the discipline;
- to develop skills in solving typical problems from the main sections of linear algebra, mathematical analysis, probability theory and elements of mathematical statistics;
- to master typical methods of mathematical research for applied issues in the field of study;
- to develop the ability to use mathematical apparatus in the study of real processes and phenomena.

Competencies and indicators formed in students:

Code and Name of Competency	Code and Name of Indicator
GPC-2 Is able to collect, process, and analyze data necessary for solving management tasks using modern tools and intelligent information and analytical systems	GPC-2.1 Possesses skills in applying mathematical apparatus

GPC (*General Professional Competence*)

2. Place of the Discipline in the Educational Program Structure

Subsequent disciplines (modules), internships, research, etc.:

- Fundamentals of Statistics,
- Concepts of Modern Natural Science,
- Fundamentals of Project Activities,
- Introduction to Professional Activities,
- Economic and Mathematical Modeling,
- Software and Programming in Professional Activities,
- Logistics,
- Accounting and Analysis,
- Risk Management,
- Internship (Work Experience).

3. Structure and Content of the Discipline

3.1. Structure of the Discipline

For Full-Time Education

Type of Academic Work	Total Credits	Total Hours	Semester 1
TOTAL WORKLOAD OF THE DISCIPLINE	7	252	252
CONTACT WORK	-	139	139
CLASSROOM WORK	3,33	120	120
Lectures	1,44	52	52
Practical (Seminar) Classes	1,89	68	68
Laboratory Work	-	-	-
INDEPENDENT WORK OF THE STUDENT	3,67	132	132
Studying Educational Material	2,67	96	96
Course Project	-	-	-
Course Paper	-	-	-
Preparation for Intermediate Assessment	1	36	36
Intermediate Assessment:			Exam -

For Part-Time (Evening) Education

Type of Academic Work	Total Credits	Total Hours	Semester 2
TOTAL WORKLOAD OF THE DISCIPLINE	7	252	252
CONTACT WORK	-	79	79
CLASSROOM WORK	1,39	50	50
Lectures	0,56	20	20
Practical (Seminar) Classes	0,83	30	30
Laboratory Work	-	-	-
INDEPENDENT WORK OF THE STUDENT	5,61	202	202
Studying Educational Material	5,36	193	193
Course Project	-	-	-
Course Paper	-	-	-
Preparation for Intermediate Assessment	0,25	9	9
Intermediate Assessment:			Exam -

3.2. Content of the Discipline, Structured by Sections and Types of Classes

Sections of the Discipline	Total Hours	Distribution of Workload by Type of Academic Work				Forms of Control	Codes of Formed Competency Indicators
		Lec.	Lab.	Pr.	Self-study		
Section 1. Introduction to Linear Algebra and Analytic Geometry	66	16		18	32	FA1	GPC-2.1 KAS
Section 2. Introduction to Mathematical Analysis	102	24		34	44	FA2	GPC-2.1 KAS
Section 3. Introduction to Probability Theory and Mathematical Statistics	48	12		16	20	FA3	GPC-2.1 KAS
Exam	36				36	**SA 1**	**GPC-2.1 KAS**
Total for Semester 1	**252**	**52**		**68**	**132**		
TOTAL	**252**	**52**		**68**	**132**		

FA (*Formative Assessment*), SA (*Summative Assessment*)

3.3. Content of the Discipline

Section 1. Introduction to Linear Algebra and Analytic Geometry

Topic 1.1. Matrices and Determinants

Concept of a matrix. Operations on matrices, properties of matrix operations. Elementary transformations of matrices.

Determinant of a matrix. Properties of determinants. Methods for calculating determinants. Expansion formula by row (by column). Rank of a matrix.

Inverse matrix. Condition for the existence of an inverse matrix. Methods for finding the inverse matrix.

Topic 1.2 Systems of Linear Algebraic Equations

Systems of linear equations. Kronecker-Capelli theorem (also known as the Rouché-Capelli theorem). Solving systems of linear equations using Cramer's rule and the matrix method. Solving systems of linear equations using the Gaussian method.

Topic 1.3 Vectors

Concept of a vector. Coordinates of a vector, length of a vector. Collinear vectors, condition for collinearity. Coplanar vectors. Operations on vectors in coordinate form. Projection of a vector onto an axis. Properties of projection. Dividing a segment in a given ratio, coordinates of the division point.

Scalar product of vectors, properties of the scalar product. Vector product of vectors, properties of the vector product. Scalar triple product of vectors, properties of the scalar triple product.

Topic 1.4 Analytic Geometry

Cartesian coordinate system. Transformations of the coordinate system. Polar coordinate system.

Equation of a line in a plane. Angle between lines. Distance from a point to a line. Equation of a plane in space. Distance from a point to a plane. Equation of a

line in space. Angle between lines. Angle between a line and a plane. Second-order curves (conic sections). Canonical form of second-order curves.

Section 2. Introduction to Mathematical Analysis

Topic 2.1 Number Sets

Concept of a set. Various types of number sets. Interval. Neighborhood of a point. Cardinality of a set. Mappings: surjection, injection, bijection. Concept of a function, graph of a function. Inverse function. Basic elementary functions, their graphs and properties.

Topic 2.2 Theory of Limits

Number sequence. Limit of a number sequence. Properties of the limit of a number sequence.

Limit of a function at a point. Properties of the limit of a function. The first remarkable limit, the second remarkable limit. Equivalences. One-sided limits.

Definition of a function continuous at a point. Discontinuity points of a function. Properties of functions continuous on an interval.

Topic 2.3 Derivative of a Function of One Variable

Concept of the derivative of a function at a point. Derivatives of basic elementary functions. Rules of differentiation.

Derivative of a composite function, logarithmic derivative. Derivative of a function defined parametrically. Derivative of an implicitly defined function. Derivative of an inverse function. Higher-order derivatives

Topic 2.4 Applications of the Derivative

Geometric meaning of the derivative.

Extrema of a function. Intervals of monotonicity of a function. Necessary condition for an extremum of a function, sufficient condition for an extremum of a function. Convexity, inflection points. Necessary and sufficient conditions. Asymptotes of the graph of a function. L'Hopital's rule.

Topic 2.5 Indefinite Integral

Antiderivative of a function. Concept of an indefinite integral. Properties of the indefinite integral. Methods of calculation.

Topic 2.6. Definite Integral

Concept of a definite integral. Properties. Newton-Leibniz formula. Change of variable in a definite integral.

Applications of the definite integral: area of a curvilinear trapezoid, arc length, volume of a solid of revolution.

Topic 2.7 Functions of Several Variables

Concept of a function of several variables (FSV). Domain of a FSV. Level curves. Partial derivatives of a FSV. Higher-order partial derivatives.

Extrema of a FSV. Necessary and sufficient conditions. Tangent plane and normal to a surface. Gradient. Directional derivative. Least squares method.

Topic 2.8 Differential Equations (DE)

Concept of a DE. Order of a DE, general solution, particular solution. First-order DE. Linear second-order DE with constant coefficients.

Topic 2.9 Series

Number series. Convergence of a number series. Properties of convergent number series. Convergence tests.

Power series. Radius of convergence, interval of convergence. Expansion of functions into a power series.

Section 3. Introduction to Probability Theory and Mathematical Statistics

Topic 3.1 Algebra of Events

Elements of combinatorics. Random event. Sample space. Operations on events.

Classical and geometric definitions of probability. Properties of probability.

Addition theorem. Conditional probability. Multiplication theorem. Total probability formula, Bayes' formula, Bernoulli's formula. Most probable number of successes. Approximate formulas: Poisson formula, local Moivre-Laplace theorem, integral Moivre-Laplace theorem.

Topic 3.2 Random Variables

Discrete random variable, continuous random variable. Distribution law of a random variable. Distribution function. Probability density function of a continuous random variable. Numerical characteristics of random variables.

Basic distribution laws for discrete and continuous random variables and their numerical characteristics. Normal distribution.

Topic 3.3 Introduction to Mathematical Statistics

Main tasks of mathematical statistics. Population and sample. Statistical distribution law. Graphical representation of a sample. Point estimates of unknown distribution parameters.

3.4. Thematic Plan of Practical Classes

Section 1. Introduction to Linear Algebra and Analytic Geometry

1. Operations on matrices. Properties of matrix operations.
2. Concept of a determinant, calculation methods. Finding the rank of a matrix.
3. Solving systems of linear algebraic equations using Cramer's rule. Finding the inverse matrix. Solving systems of linear equations using the inverse matrix.
4. Solving systems of linear equations using the Gaussian method.
5. Calculating the length and direction of a vector. Calculating projection. Scalar product of vectors.
6. Vector and scalar triple products of vectors.
7. Various forms of the equation of a line in a plane.
8. Equation of a plane in space. Equations of a line in space.
9. Second-order curves (conic sections).

Section 2. Introduction to Mathematical Analysis

10. Calculating limits of functions at infinity and at a finite point. Resolving indeterminacies.
11. The first and second remarkable limits and their consequences.
12. Rules of differentiation. Derivative of a composite function. Logarithmic derivative. Approximate calculations.
13. Higher-order derivatives. Calculating limits using L'Hopital's rule.
14. Finding extrema of functions. Finding the maximum and minimum values of a function on a set. Applying derivatives to study functions and construct

graphs.

15. Basic integration methods (direct integration, change of variable, integration by parts).
16. Integration of rational functions.
17. Integration of trigonometric functions. Integration of irrational functions.
18. Methods for calculating definite integrals. Applications of the definite integral for calculating areas, volumes, and arc length of a curve.
19. Domain of a function of several variables (FSV). Level curves. Partial derivatives of a function of several variables. Derivative of an implicitly defined function.
20. Higher-order partial derivatives. First and second order total differential. Equation of the tangent plane and normal to a surface. Directional derivative. Gradient.
21. Extremum of a function of many variables. Problems of finding maximum and minimum values in a bounded region.
22. First-order differential equations (separable equations, homogeneous, linear nonhomogeneous, Bernoulli equations, exact equations).
23. Linear second-order differential equations with constant coefficients. Method of undetermined coefficients. Method of variation of parameters.
24. Convergence tests for positive series. Alternating series.
25. Power series. Radius of convergence of a power series.
26. Expansion of functions into a power series. Application of power series for approximate calculations.

Section 3. Introduction to Probability Theory and Mathematical Statistics

27. Elements of combinatorics. Random events. Operations on events. Classical and geometric definition of probability. Addition and multiplication theorems of probability.
28. Operations on events. Total probability formula. Bayes' formula.
29. Bernoulli trial scheme. Bernoulli's formula. Limit theorems in the Bernoulli scheme.
30. Discrete random variables. Distribution function of random variables and its properties. Basic distribution laws for discrete random variables.
31. Continuous random variables. Probability density function of a continuous random variable and its properties.
32. Numerical characteristics of random variables and their properties. Examples of main distributions of random variables.
33. Population and sample. Statistical distribution series. Graphical representation of a sample: histogram, frequency polygon.
34. Processing experimental results, determining point estimates of numerical characteristics and distribution parameters of the population.

3.5. Thematic Plan of Laboratory Work

This type of work is not provided for by the curriculum.

3.6. Course Project / Course Paper

This type of work is not provided for by the curriculum.

4. Assessment of Learning Outcomes

Assessment of learning outcomes in the discipline is carried out within the framework of formative assessment and summative assessment, conducted according to a point-rating system (PRS).

Scale for assessing learning outcomes in the discipline:

Competency Code	Competency Indicator Code	Planned Learning Outcomes for the Discipline	Level of Formation of the Competency Indicator			
			High	Intermediate	Below Intermediate	Low
			from 85 to 100	from 70 to 84	from 55 to 69	from 0 to 54
			Assessment Scale			
			Excellent	Good	Satisfactory	Unsatisfactory
			Passed			Failed
GPC-2 Is able to collect, process, and analyze data necessary for solving management tasks using modern tools and intelligent information and analytical systems	GPC-2.1 Possesses skills in applying mathematical apparatus	Know:				
		concept of a matrix, definition of an inverse matrix, concept of matrix rank; concept of a matrix determinant, properties of determinants; general form of a SLAE; Cramer's rule for solving SLAEs; matrix method for solving SLAEs; criterion for consistency of a SLAE; Gaussian method; concept of a vector; various forms of equations of lines in a plane; various forms of equations of a plane in space; methods for defining a line in space; equations of second-order curves; concept of the limit of a sequence and a function at a point; concept of continuity of a function at a point and on an interval; concept of derivative, its geometric meaning; concept of indefinite and definite integrals, their properties; concept of a function of several variables, partial derivatives of a function	fluently and fully knows	knows sufficiently fully	describes poorly, makes many mistakes	does not know

		of several variables; concept of a differential equation, main types of first-order differential equations; concept of numerical and power series; basic tests for convergence of numerical series; basic concepts and definitions of probability theory and mathematical statistics: classical and geometric definitions of probability, basic formulas; concepts of discrete and continuous random variables, population and sample, point estimates of distribution parameters.				
		Be able to:				
		perform operations on matrices; find the matrix inverse of a given one; find the rank of a matrix; write a system of linear equations in matrix form; apply various methods for solving SLAEs; calculate scalar, vector, and scalar triple products of vectors; construct equations of a line in a plane and space, equations of a plane in space, second-order curves; find the angle between lines, planes, distance from a point to a line and plane; find the domain of a function, investigate a function for symmetry; calculate the limit of a sequence and a function at a point; calculate derivatives of elementary functions; find extrema of a function, the largest and smallest value of a function on a set; calculate antiderivatives of functions (in simple cases); calculate definite integrals; apply integrals to solve simple	perform s without errors	performs with minor errors	performs with a large number of errors	not able

		applied problems; find partial derivatives of a FSV, higher-order partial derivatives, extrema of a FSV; investigate numerical series for convergence; find the radius of convergence of a power series; formulate the mathematical statement of typical word problems; construct mathematical models of simple problems from real processes and conduct their analysis; choose a method for solving a typical problem; solve standard problems using formulas from probability theory and mathematical statistics.				
		Possess:				
		skills in using mathematical symbolism to express quantitative and qualitative relationships of objects; skills in symbolic transformations of mathematical expressions; ability to formulate the mathematical statement of a typical word problem; skills in constructing graphs of elementary functions; skills in using graphs and tables when solving a problem and analyzing the found solution; basic analytical methods for solving algebraic equations and systems of algebraic equations; basic methods of differentiation and integration of functions; basic methods for finding extrema of functions of one and several variables; methods for statistical processing of experimental results;	possesses fully	possesses sufficiently	possesses weakly, makes many mistakes	does not possess

		methods for analyzing and presenting statistical data.				

Assessment materials for conducting formative assessment and summative assessment are provided in the Appendix to the work program of the discipline.

A complete set of tasks and materials necessary for assessing learning outcomes in the discipline is stored at the developer's department.

5. Educational, Methodological, and Informational Support of the Discipline

5.1. Educational and Methodological Support

5.1.1. Main Literature

1. Buldyk, G. M. Collection of Problems and Exercises in Higher Mathematics: textbook for universities/ G. M. Buldyk. – 2nd ed., stereotyped. – Saint Petersburg: Lan, 2022. – 332 p. – ISBN 978-5-8114-9473-6. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/195479;
2. Ganicheva, A. V. Probability Theory: textbook/ A. V. Ganicheva. – Saint Petersburg: Lan, 2022. – 144 p. – ISBN 978-5-8114-2380-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/209762;
3. Gateliuk, O. V. Practice Book on Probability Theory and Mathematical Statistics/ O. V. Gateliuk, N. V. Manyukova. – Saint Petersburg: Lan, 2022. – 132 p. – ISBN 978-5-8114-9842-0. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/238706;
4. Gorlach, B. A. Mathematical Analysis: textbook/ B. A. Gorlach. – Saint Petersburg: Lan, 2022. – 608 p. – ISBN 978-5-8114-1428-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/211079;
5. Gorlach, B. A. Linear Algebra and Analytic Geometry: textbook for universities/ B. A. Gorlach. – 2nd ed., stereotyped. – Saint Petersburg: Lan, 2022. – 300 p. – ISBN 978-5-507-44063-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/208664;
6. Gorlach, B. A. Linear Algebra and Analytic Geometry. Practice Book for Students of Technical and Economic Specialties of Universities: textbook for universities/ B. A. Gorlach, E. P. Rostova. – Saint Petersburg: Lan, 2021. – 144 p. – ISBN 978-5-8114-6737-2. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/162373;
7. Kremer, N.Sh. Mathematics for Economists and Managers. Practice Book (for Bachelors): Textbook/ N.Sh. Kremer, B.A. Putko, I.M. Trishin, M.N. Fridman – Moscow: KnoRus, 2017. – 479 p. – (for bachelors). – ISBN 978-5-406-03462-0-M-2018. – URL: https://book.ru/book/927668 — Text: electronic;
8. Putko, B. A. Mathematics for Economists and Managers.: Textbook/ B. A. Putko, I. M. Trishin, M. N. Fridman; under the general editorship of N. Sh. Kremer. – Moscow: KnoRus, 2022. – 479 p. – ISBN 978-5-406-09054-1. – URL:

https://book.ru/book/942128 — Text: electronic.

5.1.2. Additional Literature

1. Beklemishev, D. V. Course in Analytic Geometry and Linear Algebra: textbook for universities/ D. V. Beklemishev. – 19th ed., stereotyped. – Saint Petersburg: Lan, 2022. – 448 p. – ISBN 978-5-8114-9223-7. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/189312;

2. Collection of Problems in Analytic Geometry and Linear Algebra: textbook for universities/ L. A. Beklemisheva, D. V. Beklemishev, A. Yu. Petrovich, I. A. Chubarov. – 9th ed., stereotyped. – Saint Petersburg: Lan, 2022. – 496 p. – ISBN 978-5-8114-9224-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/190976;

3. Higher Mathematics. Standard Problems with Fundamentals of Theory: textbook for universities/ A. Yu. Vdovin, L. V. Mikhaleva, V. M. Mukhina [et al.]. – 3rd ed., stereotyped. – Saint Petersburg: Lan, 2022. – 188 p. – ISBN 978-5-8114-9437-8. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/195419;

4. Gorlach, B. A. Probability Theory and Mathematical Statistics: educational and methodological manual/ B. A. Gorlach. – Saint Petersburg: Lan, 2022. – 320 p. – ISBN 978-5-8114-1429-1. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/211082;

5. Gorlach, B. A. Differentiation: textbook/ B. A. Gorlach. – Saint Petersburg: Lan, 2022. – 348 p. – ISBN 978-5-8114-2715-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/210074;

6. Ikryannikov, V. I. Higher Mathematics. Practice Book: textbook/ V. I. Ikryannikov, E. B. Shvarts; edited by V. N. Maksimenko. – Moscow: KnoRus, 2023. – 436 p. – ISBN 978-5-406-10962-5. – URL: https://book.ru/book/947632. — Text: electronic;

7. Tatarnikov, O. V. Probability Theory and Mathematical Statistics for Economists.: textbook/ O. V. Tatarnikov, E. V. Shved. – Moscow: KnoRus, 2022. – 206 p. – ISBN 978-5-406-09490-7. – URL: https://book.ru/book/943149. — Text: electronic;

8. Khusnutdinov, R. Sh. Collection of Problems for the Course in Probability Theory and Mathematical Statistics: textbook/ R. Sh. Khusnutdinov. – 2nd ed., corrected. – Saint Petersburg: Lan, 2022. – 320 p. – ISBN 978-5-8114-1668-4. – Text: electronic// Lan: electronic library system. – URL: https://e.lanbook.com/book/211733.

5.2. Informational Support

5.2.1. Electronic and Internet Resources

1. Portal "Open Education", http://npoed.ru;

2. Electronic training course "Mathematics Ek", https://lms.kgeu.ru/course/view.php?id=1093

3. Electronic training course "Mathematics (Part-time)",

https://lms.kgeu.ru/course/view.php?id=4665;

4. EBS BOOK.RU, http://book.ru

5.2.2. Professional Databases/Information Reference Systems

1. Russian National Library, http://nlr.ru/;

2. Single Window of Access to Educational Resources, http://window.edu.ru/;

3. National Electronic Library, https://rusneb.ru/.

5.2.3. Licensed and Freely Distributed Software for the Discipline

No.	Software Name	Description	Supporting Document Details
1	"IRBIS 64 (modular package): 'Reader' Workstation, 'Book Circulation' Workstation"	Library automation system meeting all international requirements for modern library systems.	Healthcare Institution "Republican Medical Library and Information Center" No. 61/2008 dated 17.06.2008. Non-exclusive right. Perpetual.
2	Windows 7 Professional (FSTEC certified)	User operating system.	"TakcNet-Service" LLC No. PO-LITS 0000/2014 dated 27.05.2014. Non-exclusive right.
3	Chrome Browser	System for searching information on the internet.	Free license. Non-exclusive right. Perpetual.
4	LMS Moodle	Software for effective online interaction between teacher and student.	Free license. Non-exclusive right. Perpetual.
5	Adobe Flash Player	Browser plug-in and runtime environment for web applications.	Free license. Non-exclusive right. Perpetual.
6	Office Professional Plus 2007 Windous32 Russian DiskKit MVL CD	Software package containing necessary office programs.	"SoftLineTrade" CJSC No. 225/10 dated 28.01.2010. Non-exclusive right. Perpetual.
7	Office Standard 2007 Russian OLP NL AcademicEdition+	Software package containing necessary office programs.	"SoftLineTrade" CJSC No. 21/2010 dated 04.05.2010. Non-exclusive right. Perpetual.

6. Material and Technical Support of the Discipline

Type of Academic	Name of Classroom,	List of Necessary Equipment and Technical

Work	Specialized Laboratory	Teaching Aids
Lectures	Classroom for lecture-type classes	Specialized educational furniture, technical teaching aids for presenting educational information to a large audience (multimedia projector, computer (laptop), screen), demonstration equipment, educational visual aids.
Practical Classes	Classroom for seminar-type classes, group and individual consultations, ongoing monitoring and intermediate assessment	Specialized educational furniture, technical teaching aids (multimedia projector, computer (laptop), screen), etc.
Independent Work	Computer lab with Internet access, room V-600a	Specialized educational furniture for 30 seats, 30 computers, technical teaching aids (multimedia projector, computer (laptop), screen), video cameras, software.
	Reading room of the library	Specialized furniture, computer equipment with Internet access and access to the Electronic Information Educational System (EIES), screen, multimedia projector, software.

7. Features of Organizing Educational Activities for Persons with Limited Health Abilities and Disabilities

Persons with limited health abilities (LHA) and disabilities have the opportunity to move freely from one educational and laboratory building to another, access all floors of educational and laboratory buildings, and study in classrooms and other premises, taking into account the peculiarities of their psychophysical development and health status.

For teaching persons with LHA and disabilities with musculoskeletal disorders, conditions for unhindered access to all educational premises are ensured. Information on the special conditions created for students with LHA and disabilities is posted on the university's website www//kgeu.ru. There is a possibility of providing technical assistance by an assistant, as well as services of sign language interpreters and sign language interpreters.

For adaptation to the perception of reference and educational material on the discipline by persons with LHA and disabilities with hearing impairments, the following conditions are ensured:

- for better orientation in the classroom, alert signals about the beginning and end of the class are used (the word "bell" is written on the board);
- the teacher attracts the attention of a hearing-impaired student with a gesture (a hand is placed on the shoulder, gentle patting is carried out);
- when talking to the student, the teacher looks at him/her, speaks clearly, in short sentences, allowing for lip reading.

Compensation for difficulties in speech and intellectual development of hearing-impaired students is carried out by:

- using diagrams, drawings, computer presentations with hyperlinks commenting

on individual components of the image;

- regularly applying exercises for graphically highlighting essential features of objects and phenomena;
- providing the student with the opportunity to receive targeted advice via email as needed.

For adaptation to the perception of reference, educational, and informational material provided for in the educational program in the chosen field of training by persons with LHA and disabilities with visual impairments, the following conditions are ensured:

- the official website on the Internet is adapted taking into account the special needs of visually impaired persons, and large-print reference information about the schedule of training sessions is provided;
- the teacher, his/her interlocutor (if necessary), and those present in the class are introduced to the students, and each time the person the teacher is addressing is named;
- actions, gestures, and movements of the teacher are briefly and clearly commented on;
- printed information is provided in large font (from 18 points) and fully voiced;
- the necessary level of illumination of the premises is ensured;
- students are given the opportunity to use computers during classes and the right to record explanations on a dictaphone (at the students' request).

The form of ongoing monitoring and intermediate assessment for students with LHA and disabilities is determined by the teacher in accordance with the curriculum. If necessary, a student with LHA or a disability, considering their individual psychophysical characteristics, is given the opportunity to undergo intermediate assessment orally, in writing on paper, in writing on a computer, in the form of testing, etc., or additional time is provided to prepare an answer.

8. Methodological Recommendations for Teachers on Organizing Educational Work with Students.

The methodological support of the student educational process is one of the determining factors for high-quality education. A university teacher, demonstrating high professionalism, erudition, a clear civic stance, self-discipline, and a creative approach to solving professional tasks, contributes to the formation of a harmonious personality during the educational process.

When implementing the discipline, the teacher may use the following methods of educational work:

- methods of forming personality consciousness (conversation, discussion, suggestion, instruction, control, explanation, example, self-control, story, advice, persuasion, etc.);
- methods of organizing activities and forming behavioral experience (task, public opinion, pedagogical requirement, assignment, training, creating educational situations, coaching, exercise, etc.);

- methods of motivating activity and behavior (approval, encouragement of social activity, disapproval, creating situations of success, creating situations for emotional and moral experiences, competition, etc.).

When implementing the discipline, the teacher must consider the following directions of educational activity:

Civic and Patriotic Education:

- formation of a holistic worldview, Russian identity, respect for one's family, society, state, spiritual, moral, and socio-cultural values accepted in the family and society, for national, cultural, and historical heritage, formation of a desire for its preservation and development;
- formation of an active civic position among students, based on traditional cultural, spiritual, and moral values of Russian society, to enhance the ability to responsibly exercise their constitutional rights and duties;
- development of legal and political culture of students, expansion of constructive participation in decision-making affecting their rights and interests, including in various forms of self-organization, self-government, and socially significant activities;
- formation of motives, moral and meaningful attitudes of the individual, allowing to resist extremism, xenophobia, discrimination based on social, religious, racial, national characteristics, interethnic and interfaith intolerance, and other negative social phenomena.

Spiritual and Moral Education:

- fostering a sense of dignity, honor and honesty, conscientiousness, respect for parents, teachers, and the elderly;
- formation of principles of collectivism and solidarity, a spirit of mercy and compassion, the habit of caring for people in difficult life situations;
- formation of solidarity and a sense of social responsibility towards people with disabilities, overcoming psychological barriers towards people with disabilities;
- formation of an emotionally rich and spiritually elevated attitude towards the world, the ability and skill to convey one's aesthetic experience to others.

Cultural and Educational Development:

- formation of an aesthetic picture of the world;
- formation of respect for the cultural values of one's native city, region, country;
- increasing the cognitive activity of students.

Scientific and Educational Development:

- formation of a scientific worldview among students;
- formation of the ability to acquire knowledge;

- formation of skills in analyzing and synthesizing information, including in the professional field.

Amendments and Approvals for the New Academic Year

No.	Section of Changes	Date of Change	Content of Changes	"Agreed" Head of the Department Implementing the Discipline	"Agreed" Chairman of the Methodological Council of the Institute (Faculty) which includes the Graduating
1	2	3	4	5	6
1	1	03.04.2024	Topic 1.4 Linear Vector Spaces was removed from the content of Section 1 "Introduction to Linear Algebra and Analytic Geometry". The number of hours for lectures (4 hours) and practical classes (4 hours) in Section 1 decreased.		
2	3	03.04.2024	The number of hours for lectures (4 hours) and practical classes (4 hours) in Section 3 "Introduction to Probability Theory and Mathematical Statistics" increased.		
3	1,3	03.04.2024	The content of the Assessment Materials (AM) was changed in accordance with the changes in the content of the discipline.		

Appendix to the course syllabus

KSPEU

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
Federal State Budgetary Educational Institution of Higher Education
«KAZAN STATE POWER ENGINEERING UNIVERSITY»
(FSBEI HE «KSPEU»)

ASSESSMENT MATERIALS
for the discipline

B1.M.12 Mathematics

(Name of the discipline in accordance with the curriculum)

Kazan, 2026

The assessment materials for the discipline are designed to evaluate learning outcomes against the indicators of competency achievement.

Assessment of learning outcomes in the discipline is carried out within the framework of formative assessment (FA) and summative assessment (SA), conducted according to a point-rating system (PRS).

1. Technological Chart

Semester 1

Section Name	Forms and Type of Control	Rating Indicators							
		I Formative	Additional Points to FA1	II Formative	Additional Points to FA2	III Formative	Additional Points to FA3	Total	Summative Assessment
Section 1. Introduction to Linear Algebra	**FA1**	**15**	**0**					**15**	**15**
Test		5	0						
Test Paper		10	0						
Section 2. Introduction to Mathematical Analysis	**FA2**			**15**	**0**			**15**	**15**
Test				5	0				
Test Paper				10	0				
Section 3. Introduction to Probability Theory and Mathematical Statistics	**FA3**					**25**	**0**	**25**	**25**
Test						5	0		
Test Paper						10	0		
Colloquium						10	0		
Summative assessment (Pass/Fail exam, Exam, Course Project/Paper)	**Exam Materials**								**0-45**
Summative assessment Task									0-15
Written form by tickets									0-30

2. Assessment Materials for Formative Assessment and Summative Assessment

Scale for assessing learning outcomes for the discipline:

Competence Code	Competence Indicator Code	Planned Learning Outcomes for the Discipline	Level of Competence Indicator Formation			
			High	Intermediate	Below Intermediate	Low
			85 to 100	70 to 84	55 to 69	0 to 54

			Assessment Scale			
			Excellent	Good	Satisfactory	Unsatisfactory
			Passed			Failed
GPC-2 Is able to collect, process, and analyze data necessary for solving management tasks using modern tools and intelligent information and analytical systems	GPC-2.1 Possesses skills in applying mathematical apparatus	Knows:				
		the concept of a matrix, definition of the inverse matrix, concept of matrix rank; the concept of a matrix determinant, properties of determinants; general form of a system of linear algebraic equations (SLAE); Cramer's rule for solving SLAEs; matrix method for solving SLAEs; the criterion for consistency of an SLAE (Rouché–Capelli theorem); Gaussian method; the concept of a vector; various types of equations of lines in a plane; various types of equations of a plane in space; methods for defining a line in space; equations of second-order curves; the concept of the limit of a sequence and the limit of a function at a point; the concept of continuity of a function at a point and on an interval; the concept of a derivative, its geometric meaning; the concept of an indefinite and definite integral, their properties; the concept of a function of several variables, partial derivatives of a function of several variables; the concept of a differential equation, main types of first-order differential equations; the concept of numerical and power series; main convergence tests for numerical series; basic concepts and definitions of probability theory and mathematical	fluently and fully knows	knows sufficiently well	describes poorly, makes many mistakes	does not know

		statistics: classical and geometric definitions of probability, basic formulas; concepts of discrete and continuous random variables, population and sample, point estimates of distribution parameters.				
		Be able to:				
		perform operations on matrices; find the inverse of a given matrix; find the rank of a matrix; write a system of linear equations in matrix form; apply various methods for solving SLAEs; calculate the scalar (dot), vector (cross), and scalar triple products of vectors; construct equations of a straight line in a plane and in space, equations of a plane in space, and second-order curves; find the angle between lines, planes, and the distance from a point to a line and plane; find the domain of a function, investigate a function for symmetry; calculate the limit of a sequence and the limit of a function at a point; calculate derivatives of elementary functions; find extrema of a function, the largest and smallest values of a function on an interval; find antiderivatives of functions (in simple cases); calculate definite integrals; apply integrals to solve simple applied problems; find partial derivatives of functions of several variables, higher-order partial derivatives, and extrema of functions of several variables; investigate numerical series for convergence; find the radius of convergence of a power	perform s without errors	performs with minor errors	performs with a large number of errors	not able

		series; formulate mathematical statements of typical word problems; construct mathematical models of simple real-world processes and conduct their analysis; choose a solution method for a typical problem; solve standard problems using formulas from probability theory and mathematical statistics.				
		Possesses:				
		skills in using mathematical symbolism to express quantitative and qualitative relationships of objects; skills in symbolic transformations of mathematical expressions; formulating the mathematical statement of a typical word problem; skills in plotting graphs of elementary functions; skills in using graphs and tables when solving a problem and analyzing the obtained solution; basic analytical methods for solving algebraic equations and systems of algebraic equations; basic methods of differentiation and integration of functions; basic methods for finding extrema of functions of one and several variables; methods for statistical processing of experimental results; methods for analyzing and presenting statistical data.	possess fully	possess sufficiently	possess poorly, makes many mistakes	does not possess

The grade "**excellent**" is given if the student completes the test papers during the semester, the test tasks, and the colloquium tasks; answers the exam ticket questions fully and with reasoned arguments; demonstrates understanding of the

material, can justify their judgments, apply knowledge in practice, and independently solve proposed tasks requiring additional analysis and the search for necessary properties and rules; presents the material consistently and correctly.

The grade "**good**" is given if the student completes the test papers during the semester, the test tasks, and the colloquium tasks; answers the exam ticket questions fully and with reasoned arguments; demonstrates understanding of the material, can justify their judgments, apply knowledge in practice, and independently solve proposed tasks requiring additional analysis and the search for necessary properties and rules; presents the material consistently and correctly, but makes 1-2 mistakes which they then correct themselves.

The grade "**satisfactory**" is given if the student completes the test papers during the semester, the test tasks, and the colloquium tasks.

The grade "**unsatisfactory**" is given for weak and incomplete completion of the test papers during the semester, test tasks, and colloquium.

3. List of Assessment Tools

Brief description of the assessment tools used in the ongoing monitoring of academic performance and intermediate certification of the student in the discipline:

Name of Assessment Tool	Brief Description of the Assessment Tool	Description of the Assessment Tool
Test Paper (TP)	A tool to test the ability to apply acquired knowledge to solve specific types of problems on a topic or section.	A set of test tasks by variant.
Colloquium (Col)	A tool to monitor the assimilation of educational material on a topic, section, or several sections of the discipline, organized as a lesson in the form of a conversation between the instructor and students.	Questions on topics/sections of the discipline.
Test	A system of standardized tasks that allows automating the procedure for measuring a student's level of knowledge and skills.	A set of test tasks.

4. List of Control Tasks or Other Materials Necessary for Assessing Knowledge, Skills, and Competencies Characterizing the Stages of Competence Formation in the Process of Mastering the Discipline

For Formative Assessment FA1:

Competence assessed: GPC-2, GPC-2.1 – Possesses skills in applying mathematical apparatus.

Test

1. Given matrix $\begin{pmatrix} 0 & 0 & 1/6 \\ 1 & 0 & 2/3 \\ -2 & 1 & -13/6 \end{pmatrix}$. What is the determinant of the inverse matrix A^{-1}?

2. The expansion of the determinant $\begin{vmatrix} -2 & 1 & 4 \\ a & b & c \\ 3 & -1 & 5 \end{vmatrix}$ along the third column has the form. Choose one answer:
$a + 2b + 5c$; $-a + 2b - 5c$; $-9a - 22b + c$; $9a - 22b - c$

3. Given matrix $A = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}$. Let the elements of the inverse matrix A^{-1} be denoted by a_{ij}^{-1}. What is the element a_{12}^{-1}? Choose one answer:
$1/3$; -1; $14/3$; $-1/3$; 0; $2/3$.

4. When solving the system $\begin{cases} x + 2y = 2, \\ 3x - 4y = 7 \end{cases}$ using Cramer's rule. Choose one answer:

$\Delta = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} 2 & 2 \\ 7 & 4 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$;

$\Delta = \begin{vmatrix} 1 & 2 \\ 3 & -4 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} 2 & 7 \\ 3 & -4 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & 2 \\ 2 & 7 \end{vmatrix}$;

$\Delta = \begin{vmatrix} 1 & 2 \\ 3 & -4 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 2 & 2 \\ 7 & -4 \end{vmatrix}$;

$\Delta = \begin{vmatrix} 1 & 2 \\ 3 & -4 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} 2 & 2 \\ 7 & -4 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$.

5. For matrices $A = \begin{pmatrix} 1 & 2 \\ -4 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ mark which operations can be performed. Choose one or more answers:
AB; BA; AB^T; A^TB; B^TA; BA^T; A^TB^T; B^TA^T.

6. Given matrices $A = \begin{pmatrix} 1 & 4 & 5 \\ -2 & 1 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 3 & 4 \\ -1 & 0 \end{pmatrix}$. Matrix $D = A + 3B^T$.

Find $d_{12} - d_{22}$.

7. If one row of a fourth-order square matrix is multiplied by 2, then the determinant...

Choose one answer:

A. increases by 16 times;

B. increases by 2;

C. decreases by half;

D. increases by half;

E. does not change.

8. Solve the equation $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 2 & 1 & 2-x \end{vmatrix} = 0$. In the answer, enter what x equals.

9. Find all values of λ, for which the inverse matrix A^{-1} exists, if the matrix

$$A = \begin{pmatrix} 9 & 8 & 7 \\ 6 & \lambda & 4 \\ 3 & 2 & 1 \end{pmatrix}.$$

Choose one answer:

$\lambda > 5$; $\lambda \in (-\infty; 5) \cup (5; +\infty)$; $\lambda \geq 5$; $\lambda < 5$; $\lambda = 5$; $\lambda \leq 5$.

10. The augmented matrix of the system is reduced to the form:

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 1 \\ 0 & -1 & 1 & 3 & 2 \\ 0 & 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Establish the correspondence between the system's characteristics (properties) and their values:

Characteristic	Value
a. the number of solutions of the system is	1. 0
b. the rank of the augmented matrix is	2. 1
c. the number of free variables is	3. 2
d. the number of unknowns in the system is	4. 3
	5. 4
	6. 5
	7. infinitely many

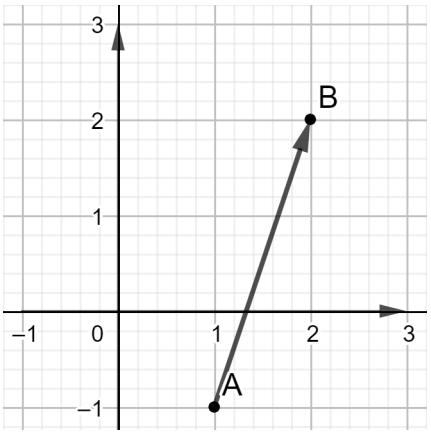
1. Find the length of the vector $a = (-6; 5; -2\sqrt{15})$.

2. For what value of α are the vectors $a = (8; -\alpha; 2)$ and $b = (-1; -2; -3)$ orthogonal?

3. The scalar (dot) product of vectors is denoted by:

$|a| \cdot |b|$; (a, b); $|a \times b|$; $a \times b$

4. Determine the coordinates of the vector $\overrightarrow{AB}$ from the figure.



5. Given points $A(1;1;1)$, $B(-2;4;0)$, $C(3;-4;5)$.
Establish the correct correspondence between operations on vectors and their coordinates:

A. $\overline{AC}-\overline{BC}$

B. $2\overline{AC}+\overline{BA}$

C. $4\overline{AB}$

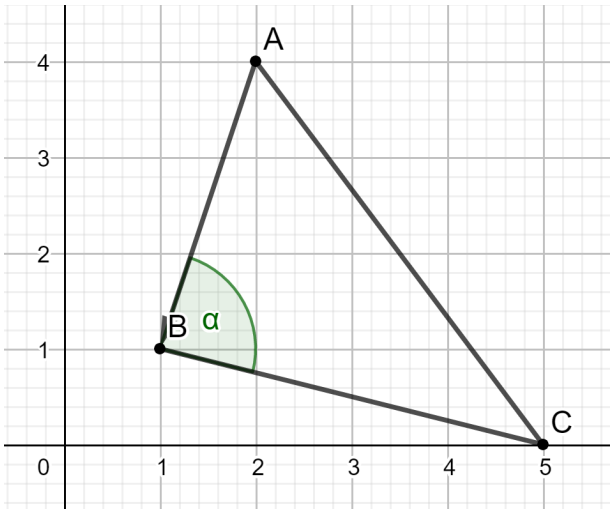
1. $(3;-3;1)$
2. $(-3;3;-1)$
3. $(-12;12;-4)$
4. $(7;-13;9)$

6. The volume of a triangular pyramid built on vectors $a=(3;3;-1)$, $b=(1;4;2)$, $c=(1;-2;0)$, is equal to. Enter the answer:

7. Given points $A(0;1;2)$, $B(-2;4;1)$, $C(1;-1;3)$. Find $np_{\overline{AB}}\overline{AC}$.

$-\frac{\sqrt{26}}{2}$ $\quad$ 13 $\quad$ $-\frac{3\sqrt{6}}{2}$ $\quad$ $-\frac{9\sqrt{14}}{14}$ $\quad$ $-\frac{3\sqrt{21}}{14}$

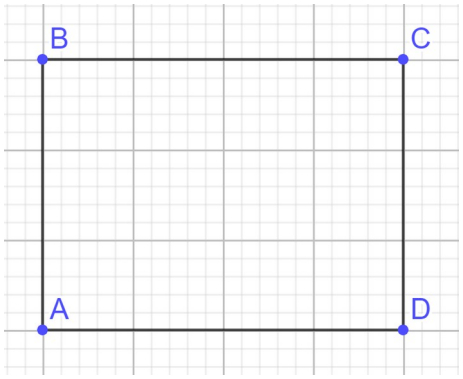
8. Given triangle ABC (see figure). Find $\cos\alpha$.



Choose one answer:

$\frac{\sqrt{3}}{3}$ $-\frac{\sqrt{3}}{2}$ $\frac{\sqrt{170}}{170}$ $\frac{9\sqrt{10}}{50}$ $-\frac{\sqrt{170}}{170}$ $-\frac{9\sqrt{10}}{50}$

9. Given rectangle $ABCD$.



Mark all pairs of collinear vectors.

$\overline{DA}+\overline{AB}$ и $\overline{BC}+\overline{CD}$;

$\overline{AD}$ и $\overline{CB}$;

$\overline{AD}-\overline{AB}$ и $\overline{DA}-\overline{DC}$;

$\overline{AC}+\overline{CD}$ и $\overline{BC}-\overline{AC}$.

10. Let φ - be the angle between non-zero vectors a and b. Establish the correspondence between the sign of $a \cdot b$ and the magnitude of the angle φ.

A. $a \cdot b > 0$

B. $a \cdot b < 0$

C. $a \cdot b = 0$

1. $\varphi = 0^\circ$
2. $\varphi = 90^\circ$
3. $0^\circ < \varphi < 90^\circ$
4. $90^\circ < \varphi < 180^\circ$

Test Paper

Variant 1.

1. Find the distance from the point $M_0(-12;7)$ to the line passing through the two points $M_1(-3;4)$ and $M_2(1;5)$.
2. Find the tangent of the angle between the line $x - 3y + 5 = 0$ and the line passing through the origin perpendicular to the vector $\bar{n}(1;-2)$.
3. Find the equation of the plane passing through the points $M_1(1,2,-1)$, $M_2(1,-2,3)$, $M_3(2,2,3)$.
4. Reduce the equation of line L: $\begin{cases} x - y - z = 0 \\ 2x + y + z - 3 = 0 \end{cases}$ to canonical form.
5. Find the coordinates of the intersection point of the line $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-2}{1}$ with the plane $3x - y + 2z + 5 = 0$.

Variant 2.

1. Find the equation of the line passing through point $M_0(-2,3)$ and perpendicular to line L: $2y - x - 8 = 0$.

2. Find the tangent of the angle between the line $x - 3y + 5 = 0$ and the line passing through the point $M_0(-2;1)$ and perpendicular to the vector $\bar{n}(1;-2)$.

3. Find the equation of the plane passing through point M(2,2,-3) and parallel to the plane $x - 4y - 2z + 1 = 0$.

4. Find the angle between line L: $\begin{cases} -x + y + 4 = 0 \\ z - 1 = 0 \end{cases}$ and plane G: $y + z - 6 = 0$.

5. Compose the equation of the line passing through the point $M_0(1,-3,5)$ parallel to the line $\begin{cases} 4x - y + 2z - 7 = 0, \\ x + 3y - 2z + 3 = 0. \end{cases}$

For Formative Assessment FA2:

Competence assessed: GPC-2, GPC-2.1 -- Possesses skills in applying mathematical apparatus.

Test

1.

Mark the correct equivalences that hold as $x \to 0$.			
$1 - \cos x \sim \frac{x^2}{2}$	$e^x - 1 \sim x$	$\arccos x \sim x$	$(1+x)^m \sim mx$

2.

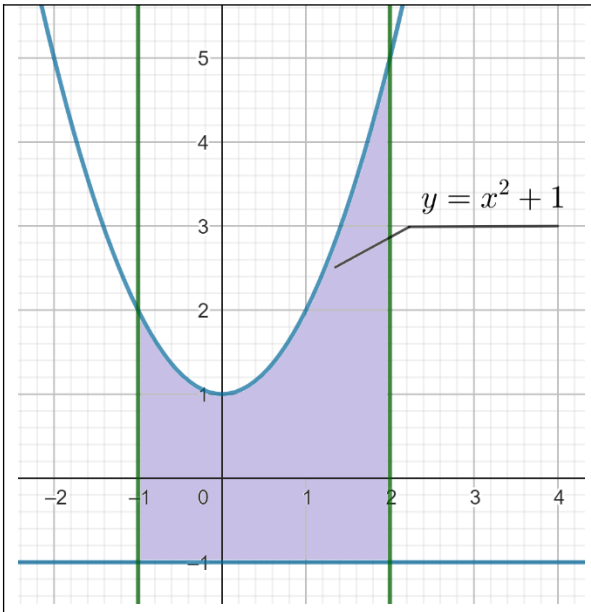
Calculate the indefinite integral $\int (3\cos x + 2x)dx$.			
* $3\sin x + x^2 + C$	$3\sin x + C$	$-\sin x + x^2 + C$	$\sin x + 2x + C$

3.

Calculate the indefinite integral $\int xe^{6x}dx$.			
$\frac{1}{36}e^{6x}(6x-1) + C$	$\frac{1}{6}e^{6x}(x-1) + C$	$6e^{6x}(x-6) + C$	$\frac{x^2}{2}e^{6x} + C$

4.

Calculate the area of the plane figure shown in the figure.



5.

Calculate the definite integral $\pi \int_0^1 x \cdot \sin(\pi x^2)\, dx$.

6.

Find the general solution of the differential equation $y'' = \cos x$.

$* -\cos x + C_1 x + C_2$	$C_1 \cos x + C_2$	$-\cos x + C$	$-\cos x + x + C$

7.

Find the general solution of the differential equation $y'' - 4y' + 4y = 0$.

$C_1 e^{2x} + C_2 x e^{2x}$	$C_1 e^{2x} + 2C_2 e^{2x}$	$e^{2x} + x e^{2x}$	$C e^{2x}$

8.

A numerical series is given $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$. Find the third term a_3 of this series.

9.

Mark the divergent numerical series.

$* \sum_{n=1}^{\infty} \frac{1}{n}$	$\sum_{n=1}^{\infty} \frac{1}{n^3}$	$\sum_{n=1}^{\infty} \frac{1}{n^2}$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

10.

Find the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x-1)^n \cdot n^2}{3^n}$$

Test Paper
Variant 1.

1. Find the derivative of the function $y = \frac{2(3x^3 + 4x^2 - 2)}{\sqrt{1+x}}$.

2. Find the derivative of the function given parametrically $\begin{cases} x = (3t^2 + 1)/3t^3, \\ y = \sin(t^3/3 + t). \end{cases}$

3. Find the derivative of the specified order $y = (4x^3 + 5)e^{2x+1}$, $y^{III} = ?$

4. Find the intervals of monotonicity and extrema of the function $y = 2\ln\left(\frac{x}{x-4}\right) - 3$.

5. Find the second-order partial derivatives of the function:
$z = x^2 \cos y + y^2 \cos x - 2xy - 3$.

6. Find the first-order total differential of the function:
$z = \ln(x^2 y + 3xy^2)$.

7. Investigate the function for extremum: $z = -x^2 y + xy^2 + 4xy$

8. Find the derivative of the function defined implicitly: $z^2 = x(y \sin x + \cos z)$.

9. Find the derivative of the function at the point M_0 in the direction of the vector $\vec{a}$: $u = \sqrt{x^2 + y^2 + z^2}, \vec{a} = i + 2j - 2k, M_0(3; -4; 5)$.

10. Compose the equation of the tangent plane and the normal line at the point $M_0(1; 2; z_0)$: $z = 3x^3 - 4y^3 + 10xy + 15$.

Variant 2.

1. Find the derivative of the function $y = \frac{\sqrt{1+x^2}}{5x^3 + x}$.

2. Find the derivative of the function given parametrically $\begin{cases} x = \sqrt{1 - t^2}, \\ y = \operatorname{tg}\sqrt{1+t}. \end{cases}$

3. HFind the derivative of the specified order $y = e^{-x} \cdot (\cos 2x - 3\sin 2x)$, $y^{III} = ?$

4. Find the intervals of monotonicity and extrema of the function $y = (2x + 3)e^{-2(x+1)}$.

5. Find the second-order partial derivatives of the function:
$z = 2xy - e^{2x} + 3x^2 y - 5$.

6. Find the first-order total differential of the function: $u = \ln\sqrt{\frac{y}{x}}$.

7. Investigate the function for extremum: $z = 2x^2 y - xy^2 + 3xy$

8. Find the derivative of the function defined implicitly: $z = xy \sin(x - 2y + z)$.

9. Find the derivative of the function at the point M_0 in the direction of the vector a: $u = 5^{xy-z} + arctg\sqrt{\frac{x-z}{y}}; \vec{a} = \vec{j} + \vec{k}; M_0(1;1;0)$.

10. Compose the equation of the tangent plane and the normal line at the point $M_0(1;2;z_0)$: $z = 4x^3 - 2y^3 + 6xy - 16$.

For Formative Assessment FA3:

Competence assessed: GPC-2, GPC-2.1 -- Possesses skills in applying mathematical apparatus.

Test

1. An event that may either occur or not occur is called

* a random event

an impossible event

a certain event

a probable event

2. Events A and B that can occur simultaneously as a result of one experiment are called

* joint

disjoint (mutually exclusive)

dependent

independent

3. The number of arrangements (permutations of size m) of *n* elements taken *m* at a time is calculated by the formula:

$\frac{n!}{(n-m)!}$; $\frac{n!}{m!(n-m)!}$; $\frac{m!}{(n-m)!}$; $(n-m)!$

4. The probability of a certain event is...

5. In how many ways can 5 books be arranged on a shelf?

6. The number of arrangements (permutations) of n elements taken m at a time with repetitions is...

n^m; $\frac{n!}{(n-m)!}$; $\frac{n!}{m!(n-m)!}$; $\frac{n!}{m!}$

7. If events A and B are mutually independent, then the conditional probability $P(A/B)$ equals...

$P(B)$; $P(A)$; $\frac{P(A)}{P(B)}$; 0

8. A coin is tossed 5 times. What is the probability that the coat of arms will appear 3 times?

9. A die is rolled. How many elements does the sample space of this experiment contain?

10. What is the probability of drawing a queen from a deck of 32 cards?

Test Paper

Problem 1. Calculate the probabilities of events using the addition and/or multiplication formulas for probabilities.

Problem 2. Calculate the probabilities of events using the total probability formula and/or Bayes' formula.

Problem 3. Calculate the probabilities of events using the Bernoulli formula, its corollaries, or its asymptotic approximations.

Problem 4. The distribution law of a discrete random variable X is given. Find the mathematical expectation (mean) and variance. Plot the graph of the cumulative distribution function of the random variable X.

Problem 5. A continuous random variable X is given by its cumulative distribution function. It is required to: a) find the probability density function; b) find the mathematical expectation (mean), variance, standard deviation; c) plot the graph of the probability density function.

Problem 6. Based on the sample data, compile a discrete statistical distribution. Find unbiased estimates of the mathematical expectation (mean) and variance of the general population, and construct the empirical distribution function.

Problem 7. Based on the sample data, compile a statistical distribution. Construct a frequency histogram and a cumulative frequency graph. Calculate unbiased point estimates of the parameters of the general population.

Variant 1

1. A device consists of 3 elements operating independently. The probabilities of failure-free operation (during time t) of the first, second, and third elements are respectively 0.6, 0.7, 0.8. Find the probability that during time t only two elements will operate without failure.

2. The first box contains 20 parts, of which 15 are standard. The second box contains 30 parts, of which 24 are standard. The third box contains 10 parts, of which 6 are standard. Find the probability that a randomly selected part is standard.

3. If, on average, left-handers make up 1%, what is the probability that among 200 people there will be four left-handers? What is the probability of finding at least 4 left-handers among 200 people?

4.
$$\begin{pmatrix} X & -7 & -4 & 3 & 6 & 7 & 9 \\ p & 0.45 & 0.1 & 0.2 & 0.05 & 0.1 & 0.1 \end{pmatrix}$$

5.
$$F_X(x) = \begin{cases} 0, & x \le 1, \\ x - 1, & 1 < x \le 2, \\ 1, & x > 2. \end{cases}$$

6. 4; 4; 3; 5; 5; 4; 3; 4; 3; 5; 3; 4; 4; 5; 6; 3; 5; 5; 3; 4.

7. 12; 14; 8; 3; 9; 6,5; 7; 7; 6; 6; 13,6; 4,8; 12,7; 6,2; 9,3; 6,1; 2,9; 3,7; 5; 4

8,4; 5,9; 10,4; 3,8; 2; 5,4; 7,6; 3,9; 6; 11,4; 3; 6; 3; 3; 3; 4,9; 10,1; 9,5; 7,1; 4,1

8,8; 6,2; 9,6; 10,1; 10,8; 5,9; 13,6; 12,9; 12,4; 8,3

Variant 2.

1. Two fighter jets simultaneously attack a bomber. The probability of one fighter hitting the bomber is 0.5; the second is 0.4. Find the probability of one hit and the probability of two hits.

2. A person, lost in the forest, came out into a clearing from which 5 roads led. It is known that for different roads, the probabilities of exiting the forest within an hour are respectively: 0.6, 0.3, 0.2, 0.1, 0.1. Determine the probability that the lost person took the first road, given that they exited the forest within an hour.

3. A certain family has 8 children. The probability of having a boy or a girl is 0.5. Find the probability that a) there are 4 boys and 4 girls; b) the number of boys is between 2 and 6 (inclusive).

4.
$$\begin{pmatrix} X & -5 & 2 & 6 & 7 & 9 & 11 \\ p & 0.4 & 0.3 & 0.1 & 0.05 & 0.05 & 0.1 \end{pmatrix}$$

5.
$$F_X(x) = \begin{cases} 0, & x \le 0, \\ \dfrac{x^2}{9}, & 0 < x \le 3, \\ 1, & x > 3. \end{cases}$$

6. 12; 13; 10; 11; 12; 9; 11; 13; 14; 15; 15; 12; 12; 10; 11; 9; 8; 7; 7; 7.

7. 1; 5; 5; 14; 12,3; 9,1; 10; 4,7; 5,1; 5,3; 4,9; 3,8; 10,7; 11,3; 11; 5,8; 4,5; 2,6; 7,4; 5,8

3,1; 4,5; 7,7; 8,3; 8; 9,2; 4; 9,4; 6,7; 6; 8; 8,4; 3,7; 2,9; 4,1; 7,3; 5; 8,2; 9; 10

11; 13,5; 12,9; 12,5; 11,6; 11,4; 10,1; 12,8; 5,1; 9.

Colloquium.

Theoretical questions for the colloquium:

1. Matrix, matrix dimensions. Linear operations on matrices.
2. Matrix multiplication. Transposition.
3. Determinant of a matrix. Properties of determinants.
4. Cramer's method for solving systems of linear equations.
5. Direction and length of a vector.
6. Collinear vectors, coplanar vectors.
7. Projection of a vector onto an axis (definition, formula for calculating the projection, properties).
8. Scalar (dot) product of vectors, its properties. Angle between vectors (formula).

9. Vector (cross) product of vectors, its properties. Formula for calculating the vector product, calculating areas.
10. Scalar triple product of vectors, its properties. Formula for calculating the scalar triple product, calculating volumes.
11. Limit of a numerical sequence, limit of a function.
12. Remarkable limits.
13. Equivalences (of infinitesimal functions).
14. Derivative of a function of one variable.
15. Rules of differentiation.
16. Table of derivatives of basic elementary functions.
17. Differentiation of composite functions.
18. Applications of the derivative: finding intervals of monotonicity of a function, finding extrema, equation of the tangent to the graph of a function.
19. Indefinite integral.
20. Table of antiderivatives of basic elementary functions.
21. Properties of the indefinite integral.
22. Integration by parts formula.
23. Definite integral.
24. Properties of the definite integral.
25. Calculation of areas of plane figures, calculation of arc length using the definite integral.
26. Functions of several variables.
27. Calculation of partial derivatives.
28. Gradient.
29. Basic formulas of probability theory.
30. Discrete and continuous random variables. Basic distribution laws and their numerical characteristics.

Test tasks for the colloquium.

1.

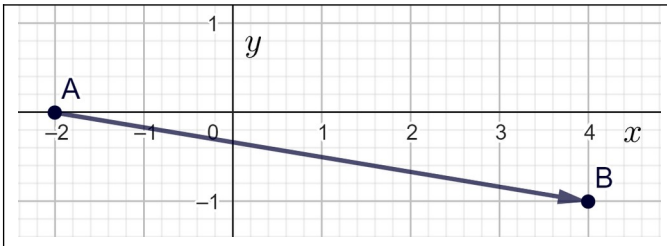
Given points $A(-1;4;6), B(0;1;5), C(-3;1;1)$. Calculate the scalar (dot) product $\overrightarrow{AB} \cdot \overrightarrow{CB}$.

2.

Given vectors $\vec{a} = \vec{i} + \vec{j} - \vec{k},\ \vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$. Calculate $\lvert\vec{b} - \vec{a}\rvert$.

3.

Find the coordinates of the vector $\overrightarrow{AB}$.



* $(6; -1)$	$(2; -1)$	$(-1; 6)$	$(4; 1)$

4.

Calculate the determinant $\begin{vmatrix} -2 & 6 \\ 1 & 5 \end{vmatrix}$.

5.

Given matrices $A = \begin{pmatrix} 1 & -3 \\ 0 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 6 \\ 1 & 7 \end{pmatrix}$. Calculate $2A + B$.

* $\begin{pmatrix} -3 & 0 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} -8 & 6 \\ 2 & 6 \end{pmatrix}$	$\begin{pmatrix} -4 & 3 \\ 1 & 3 \end{pmatrix}$	$\begin{pmatrix} 7 & -12 \\ -1 & -15 \end{pmatrix}$

6.

Mark the correct equivalences that hold as $x \to 0$.

$\sin x \sim x$	$\ln x + 1 \sim x$	$\cos x \sim x$	$(1 + x)^m - 1 \sim mx$

7.

Mark the correct formulas C – constant.

$(f \cdot g)' = f'g - fg'$	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$(C \cdot f)' = f'$	$(C)' = 0$

8.

Mark all the correct formulas.

$(\operatorname{arctg} x)' = \frac{1}{(\cos x)^2}$	$(\ln x)' = \frac{1}{x}$
$(\arcsin x)' = -\frac{1}{\sqrt{1-x^2}}$	$(a^x)' = a^x$

9.

Calculate the derivative of the function $y = \cos(5x^2 - 10)$.

$-\sin(10x - 1)$	$-\sin(10x)$	$-10x \cdot \sin(5x^2 - 10)$	$-\sin(10x - 10)$

10.

Calculate the derivative of the function $y = x^3 \cdot \sin x$.

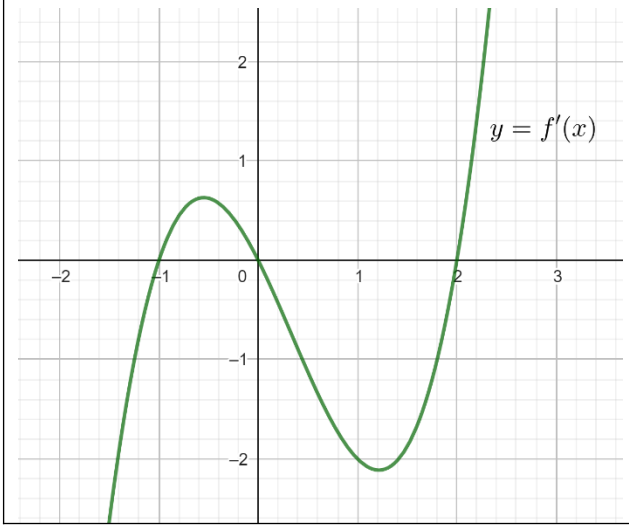
$x^2(3\sin x + x\cos x)$	$3x^2 \cos x$	$-3x^2 \cos x$	$x^2(3\sin x - x\cos x)$

11.

Find the value of the function $y = \frac{4x-12}{x^2-8}$ at the point of local minimum.

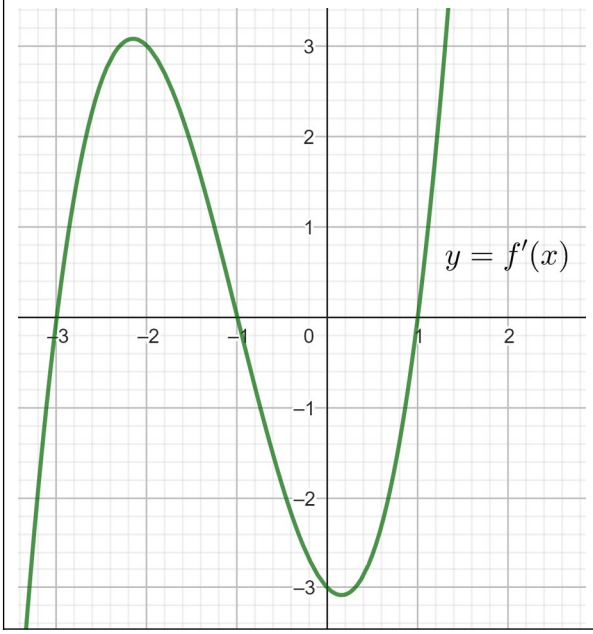
12.

The figure shows the graph of the derivative of the function $y = f(x)$. Determine from the graph the number of critical points of the function $y = f(x)$.



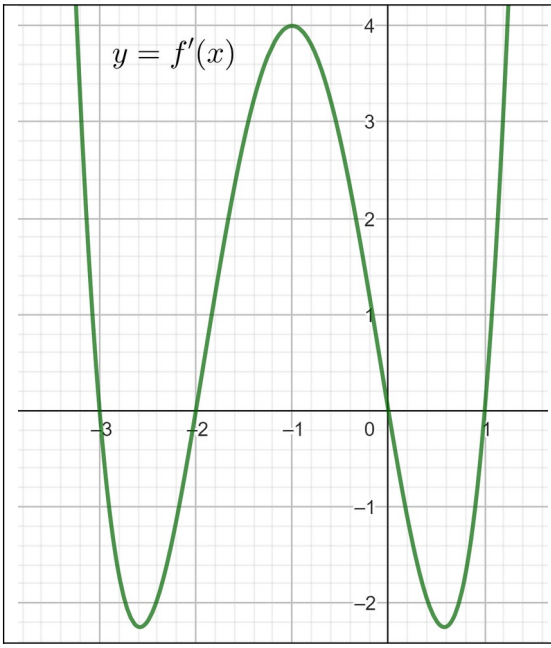
12.

The figure shows the graph of the derivative of the function $y = f(x)$. Write down the intervals of increase of the function $y = f(x)$.



13.

The figure shows the graph of the derivative of the function $y = f(x)$. Write down the intervals of decrease of the function $y = f(x)$.



14.

Mark all the correct formulas.	
$\int x^n dx = n \cdot x^{n-1} + C$	$\int \cos x \, dx = \sin x + C$
$\int \ln x \, dx = \frac{1}{x} + C$	$\int \frac{dx}{(\cos x)^2} = \operatorname{tg} x + C$

15.

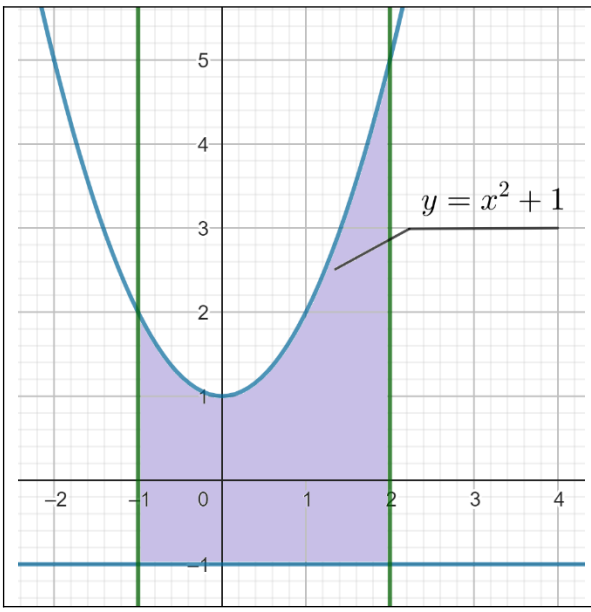
Calculate the indefinite integral $\int (6x^2 + 3)dx$.			
$6x^3 + C$	$2x^3 + 3x + C$	$6x^3 + 3x + C$	$2x^3 + C$

16.

Mark all the correct properties of the definite integral.
$\int_a^b f(x)dx = \int_b^a f(x)dx$
$\int_a^b f(x)dx = \int_a^c f(x)dx - \int_c^b f(x)dx$, $c \in (a; b)$
$\int_a^b C \cdot f(x)dx = C \cdot \int_a^b f(x)dx$, $C - const$
$\int_a^a f(x)dx = 0$

17.

Calculate the area of the plane figure shown in the figure.



18.

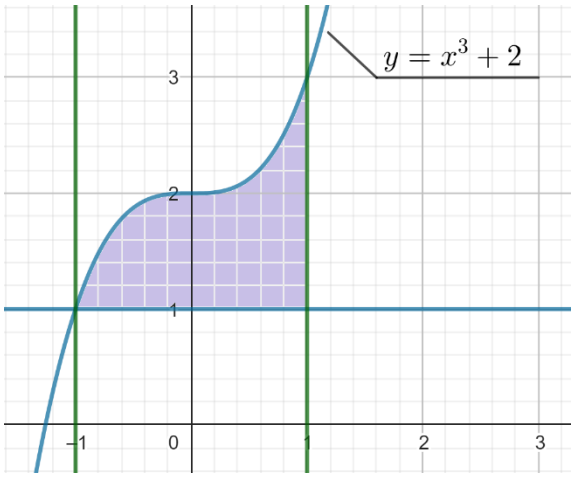
Find the critical point of the function $z = x^2 + xy + y^2 - 3x - 6y + 9.$

19.

Find the partial derivative with respect to the variable of the function
$z = x^3y + e^y + \cos(3x + 5y).$

20.

Calculate the area of the plane figure shown in the figure.



For Summative Assessment:

Theoretical questions for the exam:

Matrices and Determinants

Concept of a matrix. Matrix dimensions. Diagonal matrix. Identity matrix. Operations on matrices. Properties. Rule of matrix multiplication. Matrix transposition. Properties of matrix operations.

Determinants. Order of a determinant. Formula for expansion by a row (column). Properties of determinants. Non-singular matrix. Calculation of the inverse matrix (by algorithm, Jordan-Gauss method). Rank of a matrix (two methods for calculating matrix rank).

Systems of Linear Algebraic Equations

Systems of linear algebraic equations. Coefficient matrix. Consistent and inconsistent systems. Kronecker-Capelli theorem. Cramer's method, cases where it is applicable. Matrix method, cases where it is applicable. Gaussian method.

Analytical Geometry

Rectangular (Cartesian) and polar coordinate systems. Formulas relating rectangular and polar coordinates.

Equation of a line in a plane, main types of equations: general equation, equation with slope, canonical equation, equation of a line passing through two given points, equation of a line in intercept form (it is necessary to write the equations themselves and the meaning of their parameters). Angle between lines (formulas for calculation), special cases. Formula for calculating the distance from a point to a line in a plane.

Equation of a plane in space. Normal vector. Main types of plane equations: general equation, equation of a plane passing through three given points, equation of a plane passing through a given point perpendicular to a given vector, equation of a plane in intercept form, normal equation of a plane (it is necessary to write the equations themselves and the meaning of their parameters). Angle between planes, special cases. Distance from a point to a plane.

Equation of a line in space. Direction vector. Main types of equations of a line in space: canonical equation, parametric equations, equation of a line passing through two given points, general equations of a line (it is necessary to write the equations themselves and the meaning of their parameters). Angle between lines in space, angle between a line and a plane.

Second-order curves. Canonical second-order curves: circle, ellipse, hyperbola, parabola (derivation of equations).

Limit of a Function

Limit of a function at a point. The first remarkable limit. The second remarkable limit. Equivalent infinitesimal functions: definition, basic equivalences.

Derivative of a Function

Definition of the derivative, its geometric meaning. Formulas for calculating the derivative of the sum, difference, product, and quotient of two functions. Derivative of a composite function. Derivatives of basic elementary functions (know by heart!). Derivation of formulas for calculating the derivatives of the following functions: $x^n, \cos x, \sin x, a^x.$ Derivative of a function defined parametrically. Logarithmic differentiation. Higher-order derivatives.

Applications of the Derivative. Investigating Functions Using Derivatives

Increase and decrease of a function, necessary and sufficient conditions. Extrema of a function. Necessary condition for an extremum. Sufficient condition for an extremum of a function at a point. Concavity/Convexity, inflection points.

Indefinite Integral

Concept of an indefinite integral. Properties of the indefinite integral. Integration by parts formula. Table of integrals (know by heart!).

Definite Integral

Concept of a definite integral. Geometric meaning (area of a curvilinear trapezoid). Newton-Leibniz formula. Basic properties of the definite integral. Integration by parts formula for the definite integral.

Functions of Several Variables

Definition of a function of several variables. First-order partial derivatives. Higher-order partial derivatives. First and second-order total differential. Differentiation of an implicit function. Extrema of functions of two variables. Necessary condition for an extremum (formulation), sufficient condition for an extremum (formulation).

Differential Equations

General form, general solution, general integral. Particular solution. Cauchy problem. *First-order DE*: equations with separable variables, homogeneous differential equations, linear equations, Bernoulli equations, exact differential equations (canonical form, methods for solving each type of equation).

Linear homogeneous differential equations of the second order. Structure of the general solution of a second-order linear homogeneous DE with constant coefficients. Characteristic equation, form of the general solution for each case ($D > 0,\ D = 0,\ D < 0$). General form of a second-order linear non-homogeneous DE. Structure of the general solution of a second-order linear non-homogeneous DE. Superposition principle.

Probability Theory

Sample space. Disjoint (mutually exclusive) events. Operations on events.

Definitions of probability: classical, geometric. Properties of probability.

Conditional probability. Multiplication theorem. Total probability formula. Bayes' formula. Bernoulli trials scheme. Bernoulli formula.

Discrete and continuous random variables. Distribution of random variables. Distribution function (CDF) for discrete and continuous random variables. Formula for calculating the probability of a random variable falling into an interval. Probability density function (PDF) of a continuous random variable. Properties of the density function.

Mathematical expectation (mean). Properties of mathematical expectation. Variance. Properties of variance. Standard deviation. Formulas for calculating mathematical expectation and variance for discrete and continuous random variables.

Mathematical Statistics

Sample. Statistical distribution. Variation series. Histogram, cumulative frequency graph. Empirical distribution function. Point estimates of distribution parameters: sample mean, sample variance (corrected, uncorrected), sample standard deviation.

Exam Problems:

Basic Difficulty:

1. Calculate the fourth-order determinant:

$$\begin{vmatrix} 2 & 3 & 4 & -1 \\ -2 & 1 & 0 & 3 \\ 1 & 3 & 5 & -6 \\ 3 & -1 & 3 & 0 \end{vmatrix}$$

2. Solve the system using the matrix method:

$$\begin{cases} 2x + y + z = 1, \\ x - 4y + 2z = -1, \\ 3x - y + 2z = 0. \end{cases}$$

3. Problems in analytical geometry on a plane and in space:

3.1 Compose the equation of the line passing through the point $A(1, 3)$ parallel to the line l_1, if line l_1 passes through the points $M_1(-3, 5), M_2(1, -2)$.

3.2 Compose the equation of the plane passing through the points $A(1, 3, 0), B(-2, 4, 1), C(2, 0, -3)$.

3.3 Find the intersection point of line l and plane Ω

$$l: \frac{x-1}{5} = \frac{y+2}{3} = \frac{z-1}{1},$$
$$\Omega: 3x - 4y + z - 5 = 0.$$

3.4 Compose the canonical equations of line l, given by general equations:

$$l: \begin{cases} -x + y = 1, \\ 5x + y - z + 2 = 0. \end{cases}$$

3.5 Compose the parametric equations of the line passing through point $A(-1, 1, -1)$ parallel to line l_1, if

$$l_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{-1}.$$

4. Differentiation of a function of one variable

4.1. Find the derivative of the composite function y':

$$y = \cos\left(3x^2 + e^x\right); \quad y = \arcsin\sqrt{3x - 1}; \quad y = \frac{\sin x}{\left(x^2 + e^x\right)^3}; \quad y = \log_4 x \cdot \left(5 + \sqrt{e^x}\right).$$

4.2. Find the derivative of the function y'_x, given parametrically:

$$\begin{cases} x = \cos^3 t, \\ y = \sin^3 t; \end{cases} \quad \begin{cases} x = \ln t, \\ y = \sqrt{t}; \end{cases} \quad \begin{cases} x = arctgt, \\ y = \dfrac{1}{1+t^2}. \end{cases}$$

5. Partial derivatives of functions of several variables

5.1. Find the partial derivatives of the function:

$z = x^3 + 4x^2y - y^3$, $z = \ln(x^2 + y^2)$, $z = (x^3 - 2y^4)^3$, $z = \sqrt{1 - 4x^2 + 10y^3}$,

$z = \sin^2(8x - 2y)$, $z = \frac{y}{x}$.

5.2. Find the partial derivatives of the function defined implicitly:

$e^z + \cos(x^2 + y) - xyz = 0$, find z'_x, z'_y.

6. Indefinite integral

6.1. Calculate the indefinite integral of a fractional-rational function:

$\int \frac{x^2 + x + 2}{x(x-2)(x-4)} dx$; $\int \frac{x+2}{x(x^2 - 3x - 4)} dx$; $\int \frac{x+1}{x^2(x-5)} dx$; $\int \frac{x+4}{(x-1)(x+2)^2} dx$;

$\int \frac{x+5}{x^2 + 4x + 13} dx$.

6.2. Calculate the indefinite integral using integration by parts:

$\int (3x+5)\cos 2x dx$; $\int (2x-7)e^{3x} dx$; $\int (3x^2 + x)\ln x dx$.

6.3. Calculate the indefinite integral of a trigonometric function:

$\int \sin^5 x \cos^2 x dx$; $\int \sin^2 x \cos^4 x dx$.

7. For participation in student qualifying competitions, 4 students were selected from the first group of the course, 6 students from the second group, and 5 students from the third group. The probabilities that a student from the first, second, and third groups will make it to the institute's team are 0.9, 0.7, and 0.8, respectively. One of the selected students ultimately made it to the team. What is the probability that they belonged to the first group?

8. The distribution of the random variable X is given in the table. Find the distribution function (CDF), plot its graph. Find the mathematical expectation (mean) and variance of the random variable X.

X	0	1	3	4	5	6
p	0.1	0.15	0.05	0.2	0.45	0.05

Increased Difficulty:

1. Solve the system using the Gaussian method:

1.1
$$\begin{cases} 5x_1 + 4x_2 + x_3 + 3x_4 = -5, \\ 2x_1 + x_2 + x_3 + 4x_4 = 2, \\ 3x_1 + 2x_2 + x_3 + x_4 = -3, \\ x_1 + 3x_2 - 2x_3 + 2x_4 = -4. \end{cases}$$

1.2
$$\begin{cases} 3x_1 + 4x_2 + 2x_3 + x_4 = 16, \\ x_1 + 7x_2 + x_3 + 4x_4 = 23, \\ 2x_1 + x_2 + 3x_3 + 5x_4 = 10, \\ 4x_1 - 3x_2 + 4x_3 + 6x_4 = 1. \end{cases}$$

1.3 $\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = 7, \\ 2x_1 + 5x_2 + x_3 - 2x_4 = 5, \\ 3x_1 - 7x_2 + 4x_3 + 5x_4 = -11, \\ 7x_1 + 2x_2 - x_3 + 11x_4 = 6. \end{cases}$

3. Applications of the derivative of a function of one variable

Find the intervals of monotonicity, extrema, and intervals of convexity/concavity of the function: $y = 2\ln\left(\frac{x}{x-4}\right) - 3$.

4. Applications of partial derivatives

Find the extrema of the function: $z = -x^2y + xy^2 + 4xy$

5. Applications of the definite integral

5.1. Calculate the arc length of the curve given by the parametric equations

$$\begin{cases} x = 4(2\cos t - \cos 2t), \\ y = 4(2\sin t - \sin 2t), \quad 0 \le t \le \pi. \end{cases}$$

5.2. Calculate the area of the figure bounded by the graphs of the functions $y = \frac{e^{1/x}}{x^2}$, $y = 0$, $x = 2$, $x = 1$.

6. The random variable X is given by the distribution function $F_X(x)$. Find the probability density function, mathematical expectation (mean), and variance of the random variable.

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \frac{3x - x^2}{2}, & 0 < x \le 2 \\ 1, & x > 2 \end{cases} \qquad F_X(x) = \begin{cases} 0, & x \le \sqrt{2} \\ \frac{x^2}{2} - 1, & \sqrt{2} < x \le 2 \\ 1, & x > 2 \end{cases}$$