ISSN 1063-7729, Astronomy Reports, 2020, Vol. 64, No. 9, pp. 795–803. © Pleiades Publishing, Ltd., 2020.
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# Development of Methods for Navigational Referencing of Circumlunar Spacecrafts to the Selenocentric Dynamic Coordinate System

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Received December 19, 2019; revised January 24, 2020; accepted January 24, 2020

**Abstract**—The initial form of present-day space optical observations contain considerable geometrical and brightness distortions. This problem can be solved based on geometrical correction and transformation of reference object coordinates into commonly accepted cartographic projections. In this work, the method of coordinate transformation to the reference selenocentric dynamic system by using a base of reference selenographic objects as electronic maps is considered. The transformation process presupposes the formation of a photogrammetrically corrected image and identification of observed objects included into the electronic maps. Height data of on the Moon's surface are determined from known observed selenographic coordinates with respect to reference objects by software tools. Preliminary selenographic coordinates can be determined using both ground positional observations and onboard goniometric devices such as a laser interferometer.

**DOI:** 10.1134/S1063772920100017

## 1. INTRODUCTION

In the field of carrying out the system analysis of selenophysical data from present-day lunar missions, developing the method for reducing different selenophysical observations to a common selenocentric system, and creating a digital catalog of selenocentric reference points, one should note the following [1]. At present, in spite of achievements in the determination of selenophysical parameters based on measurement data from spacecrafts, the problem of creating exact coordinate–time orientation on the Moon's surface remains unsolved to a certain degree. According to [2], a conclusion exists that orbital parameters of the GRAIL and LRO lunar missions were determined and referenced to the celestial coordinate system with an accuracy of 1 m throughout the whole route of the satellite flight. This conclusion is based on that images received from the LRO board were referenced to each other with this accuracy for selenographic coordinates when every pixel corresponds to 0.5 m on the Moon's surface. It is also believed that the gravity field model constructed by GRAIL measurements is everywhere equated and matched with orbital elements of the LRO mission. However, according to [3] in which observations obtained by the LRO spacecraft are analyzed, almost all data on scanning the lunar satellite for the determination of its position in the phase space are related to results of Doppler ground tracking, carried out mainly with the use of the DSN system (NASA Deep Space Network). Referencing to the DSN system is insufficiently exact, both relative to the celestial coordinate system and to the Earth's coordinate systems. Results presented by Konopliv et al. [3] on laser altimetry of the Moon's surface and structural elements of the Moon's selenoid are also not referenced to any specific coordinate system but only oriented in a certain way with respect to the Moon's position. Each spacecraft revolution around the celestial body corresponds to a certain orbit. Based on element differences of such orbits at points at which the orbits intersect, the accuracy of coordinate–time referencing for the circumlunar satellite is estimated. In this case, the determination of an adequate internal accuracy of the orbital motion can be discussed. Using present-day altimetric space measurements and considering gravity perturbations improves the matching of orbital parameters and increases the internal accuracy. Still, this does not allow one to estimate the external accuracy of coordinate–time referencing. Therefore, the described technique can be used only in the case where the satellite orbits are referenced to each other. The LRO mission is also not an exception.

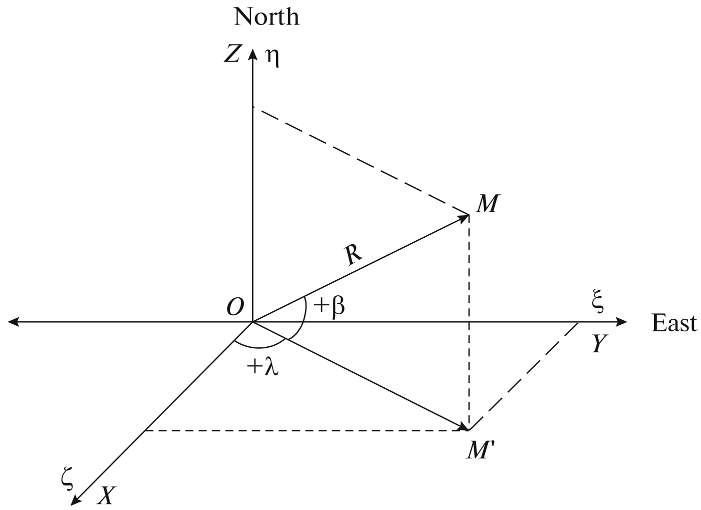


**Fig. 1.** Systems of selenographic coordinates.

Estimation of the external accuracy requires another kind of observational data whose accuracy is not worse than the internal accuracy. Referencing of the spacecraft to stars and objects on the Moon's surface can serve as such observations [4]. In this work, a method of regression modeling for reducing different selenophysical measurements to a common selenocentric system and constructing the digital selenocentric catalog (DSC) of reference lunar objects based on their direct referencing to stars and transformation of data from present-day lunar missions into the DSC system is proposed. The described results, with allowance for new theories of physical libration of the Moon, allow one to create a dynamic selenocentric reference system with coordinate axes that coincide with the lunar axes of the Moon's inertia and the coordinate origin coincides with the Moon's center of mass. It should be noted that the first attempt of navigational referencing to the digital selenographic catalog in the world practice was made by measurement systems of the LRO mission. On the platform of this spacecraft, the LALT measurement system was mounted. LALT included the LROC main optical camera allowing one to obtain lunar surface images with a resolution of 0.5 m and LOLA altimeter for the creation of an exact map of height data. The LALT system was used with the aim of determining the most suitable locations for lunar landing of modules of future lunar missions. Special attention in implementing the LRO project was directed to the increase in the accuracy of finding the satellite orbit elements.

High resolution images covering the given lunar territories were obtained using a special panoramic camera. Based on photogrammetric methods and special software, an attempt was made to reference the satellite orbit to the digital reference catalog of lunar objects. A promising direction of coordinate–time support is referencing to a system of light laser beacons (LLBs). Such system is similar to the system of reference lunar craters; however, the latter are point objects with which one can implement in principle coordinate referencing with an accuracy equal to several centimeters using the method developed in this work. It is planned to set up such beacons in the course of Russian Luna-25 (26, 27, and 28) lunar missions. Using LLBs will make it possible to implement high-accuracy lunar landing and to perform highly detailed scanning of the Moon. In the process of navigational referencing of space satellites to the lunar coordinate system, our method allows one to use both objects with known coordinates on the Moon's surface and quantum optical systems for which LLBs with determined exact coordinate positions serve as analogues.

## 2. SELENOGRAPHIC COORDINATE SYSTEM

When creating the reference catalog of lunar objects of the visible and hidden sides of the Moon (CLO), it is necessary to exactly determine the coordinate systems in which all constructions will be performed because there are certain disagreements in the special literature on this subject. The rectangular selenographic coordinates are reckoned in fractions of the Moon radius $R = 1738.1$ km.

The direction of the axes is as follows (see Fig. 1): $\zeta$ is directed to the Earth ($x$ in the Cartesian coordinate system); $\xi$ coincides with the projection of the lunar equator and is directed to the East ($y$ in the Cartesian coordinate system); and $\eta$ coincides with the projection of the zero meridian and is directed to the North Pole ($z$ in the Cartesian coordinate system).

The meridian passing through the first lunar radius, i.e., through the axis of inertia $X$, has the longitude of 0°.

The longitude is reckoned in degrees. According to classics of selenography [5, 6], the selenographic longitude of a point is equal to the dihedral angle between the zero meridian and meridian of the crater. It is positive in the eastern hemisphere and negative in the western hemisphere of the Moon. In both directions, longitudes increase to 180° in the first case and decrease to –180° in the second case. The meridian of 180° passes through the center of the hidden side of the Moon and traverses Mare Ingenii. However, according to [7], the definition is presented incorrectly—positive longitudes are reckoned to the West from the zero meridian and negative ones to the East. Therefore, one must be attentive to publications and definitions which are presented in different sources because a large number of scientific works were published later based on them.

The latitude is reckoned in degrees. The arc reckoned from the equator along the meridian passing through the crater to crater is called the latitude. It is measured from 0° to 90° to the North Pole of the Moon and from 0° to –90° to the South Pole.

The formulas of the relation between spherical and rectangular coordinates have the following form:

$$\begin{gathered}\xi = R\sin\lambda\cos\beta, \quad \eta = R\sin\beta, \\ \zeta = R\cos\lambda\cos\beta,\end{gathered} \tag{1}$$

where $\lambda$ and $\beta$ are the selenographic longitude and latitude, and $R$ is the radius vector of the lunar object.

## 3. METHOD OF CREATING THE REFERENCE CATALOG OF LUNAR OBJECTS FROM THE VISIBLE AND HIDDEN SIDE OF THE MOON

To construct the selenocentric dynamic reference system of coordinates, it is necessary to use methods of absolute referencing of point lunar objects to stars. To increase the number of reference coordinate points included in the DSC, other objects obtained from different observations are transformed into it. The method of creating the reference catalog of lunar objects of the visible and hidden sides of the Moon is based on the robust modeling algorithm. Since the transformation of selenographic positions requires a large amount of recalculations, such calculations need preliminary analysis and expediency estimation. To solve the problem, one just uses methods of regression modeling.

1. Full enumeration method of regression model elements. The method is based on polynomial expansion of coordinate data. The expansion can contain polynomials of the second and third power. Using full enumeration of terms of the regression model during the transformation of each standard coordinate, the optimum structure of the regression model with the minimum root-mean-square error (RMSE) is found. Using the method of the floating regression model allows one to increase the accuracy of the transformation of selenocentric coordinates more than by 10%.

2. Affine transformation method. This approach is used for the transformation of selenocentric coordinates from one Cartesian system $\vec{\mathbf{X}}$ to another system $\vec{\mathbf{Y}}$.

The efficiency of the procedures used in the transformation of selenocentric coordinates is determined using the exact comparative analysis. Such procedures include.

1. Optimization of polynomial approximations.

2. Affine transformations.

3. Orthogonal transformations with enumeration of systematic errors.

4. Solution of linear equation systems.

Different observations of coordinate positions were reduced to a common selenocentric system using the following method. A regression model allowing one to carry out a robust estimation of different observed data was constructed [8]:

$$X = AY + X_0 + \varepsilon, \tag{2}$$

where $X$ is the resulting matrix of different observational data reduced to a common system, $Y$ is the matrix of different observational data, $A$ is the matrix of rotation, $X_0$ is the displacement vector of the coordinate systems, and $\varepsilon$ is the error matrix. System (2) was solved using adaptive regression modeling (the ARM approach). For this purpose, the following procedures were carried out.

1. Diagnostics of qualitative components of the regression model of different data.

2. Analysis of satisfying the main conditions for the use of the least squares method (LSM) (for example, the Gauss–Markov conditions).

3. Use of numerical adaptation in the case where standard conditions of the LSM are broken.

Robust estimation of unknown parameters presupposes the following necessary procedures.

1. Criteria analysis for system transformations.

2. Creation of the spectrum of competitive regression models of reducing different data to a common system.

3. Construction of sufficient number of structured parametric model identification methods.

4. Application of different observation processing scenarios making it possible to carry out prediction estimations for qualitative properties of models and allowing one to determine condition violations for the LSM algorithm and to carry out adaptation to optimum parameters of robust estimates if a discordance with the model's postulated properties exists.

Condensation and expansion of the catalog of lunar objects are carried out with the use of the following regression model:

$$A \times \Theta + \vec{\varepsilon} = Z, \qquad (3)$$

where $A(A_{ij})$ is the rotation matrix, and $\Theta(\Delta\xi, \Delta\eta, \Delta\zeta)$ is the displacement vector of the reference system $Z(\Delta X, \Delta Y, \Delta Z)$ with respect to $\vec{\varepsilon}$.

In the course of ARM, it is necessary to consider that the transformation structure of unknown parameters for each pair of systems remains unknown during the regression model solution (3) and it is necessary to determine it according to given competitive data [9]. In the general case, expression (3) can be written as the regression matrix equation [1]:

$$Y = X\vec{\beta} + \vec{\varepsilon}, \qquad (4)$$

where $\vec{\varepsilon}$ is the error vector, and vector $\vec{\beta}$ is the first row in the rotation matrix $A$. It is evident that terms of vector $Y$ for a simple event (4) are determined simultaneously by the structure identifier.

The rotation matrix $A$ sometimes does not correspond to conditions of an orthogonal transformation from system $Y$ to system $X$ due to the presence of multicollinearity and errors in calculations of object coordinates entering into the two systems. Here, the following conditions must be satisfied:

$$A^T A = E, \quad \det A = 1. \qquad (5)$$

Equation (4) in combination with condition (5) can be interpreted as a joint deterministic transformation. Such problem is solved using the numerical optimization method which is characterized by accuracy within the difference between the $\vec{X}$ and $\vec{Y}$ systems.

The unknown parameters $\Theta(\Delta\xi, \Delta\eta, \Delta\zeta)$ are found according to the expression

$$\Theta = (A^T PA)^{-1}(A^T PZ), \qquad (6)$$

and errors for corrections to coordinates $\Delta\xi$, $\Delta\eta$, and $\Delta\zeta$ are determined according to the covariance matrix:

$$D(\Theta) = \frac{V^T PV}{2n - 3}(A^T PA)^{-1}, \qquad (7)$$

where $V$ is the vector of residual deviations.

Construction of the selenocentric catalog of reference points involves different methods for deterministic and approximate coordinate transformations. The construction process uses mathematical approaches developed for direct referencing of lunar objects to the celestial coordinate system. Solving the problem described above needs an LSM modification with allowance for the regression modeling method. This procedure requires using the algorithm of affine data transformation from one system to another [8].

The selenocentric reference coordinate system was created using data of ground observations and space missions, as well as author software packages allowing one to analyze regression models and carry out robust estimations of unknown parameters. The investigation algorithm included the following procedures.

1. Investigation of random and systematic errors of the selenocentric reference coordinate system.

2. Multiparameter harmonic and fractal analysis of selenocentric coordinates of objects on the visible and hidden sides and in the Moon's libration zone.

The catalog of reference objects was constructed using software algorithms developed for object referencing on the Moon's surface.

The reference selenocentric system was created using algorithms developed for absolute referencing of lunar objects to stars. At present, at least two of the problems solved here by use of the LSM can be solved more accurately. Also, considering that reprocessing of basic observations requires a large amount of calculations, it is necessary to determine and numerically estimate the expediency of such procedures.

One of the most important stages of the computational process during the reduction of observations is the procedure of finding (estimating) parameters of regression models which are successfully applied in astronomy, geodesy, and astrophysics. However, the classical approach to estimating unknown values in these research areas postulates a stable fixed model and the use of the LSM; therefore, it is not adequate to present-day practical requirements and development of algorithms based on regression modeling. The attempted steps for moving beyond standard LSM frames pursue local problems and do not create conditions for the system solution of this problem. The standard limitations in reducing space data and ground observations are the presence of insignificant, noninformative, doubling, and correlating polynomial terms, as well as violation of LSM conditions in regards to the normal Gauss–Markov scheme. In this work, the regression modeling approach is used as an alternative to the classical method. When estimating unknown parameters, this approach presupposes robust analysis, verification of assumptions, and adaptation to the case in which the assumptions are violated. It also requires developing a specialized software for purposes of automated data processing which reduces the time of computations and data analysis. Regression modeling is a system approach in which correctness of using any system element (structure estimation method, parameter estimation method, model, sample, set of postulations, and quality measures) can be verified and adapted if the main conditions are violated. This work involves the system approach of regression modeling and the software developed for this purpose, with analogues absent in the world scientific practice. At present, a complex of specialized programs allows one to implement an exact reduction of observations in the first approximation both for dynamic and for multifactor regressions. The software packages are as follows.

1. RASP (regression analysis software package); it is intended first of all for analyzing the optimum data processing model used for the prediction. RASP has a sufficiently rational structure including: (i) the controlling program node; (ii) the program node of query creation; (iii) the set of software function procedures; (iv) the program node of the scenario; (v) the interactive module of system settings; (vi) the information editor module; (vii) the module of creation of interactive tables; and (viii) the interactive guide.

2. MHASP (multiparameter harmonic analysis software package); the developed software package is an automated specialized product, which implements regression (robust) modeling for executing mathematical-interpretation tasks of structure parameters and anomalies of the gravity field. The main purpose of the MHASP is regression modeling of phenomena and processes with their subsequent use for predicting final values of the unknown parameters (responses).

3. SSPEGR (software system of parametric estimation of genetic relations); the developed software system is a specialized complex intended for problems of regression (robust) modeling when performing the procedure of estimating the unknown values. The main purpose of the SSPEGR is constructing multiparameter regression models (MRMs). The MRM functionality is not oriented to modeling prediction values; the main objective of the SSPEGR is describing cause-and-effect relations of processes under study.

## 4. DIGITAL CATALOG OF LUNAR OBJECTS OF THE MOON'S VISIBLE AND HIDDEN SIDES

The digital selenocentric dynamic catalog (DSDC) of lunar objects of the Moon's visible and hidden sides was constructed by data of ground observations of lunar craters when referencing to stars and Clementine, Kaguya, LRO, and Apollo satellite missions. The DSDC structure is formed by values of selenographic rectangular coordinates, selenographic longitudes and latitudes, and radius vectors of craters on the Moon's surface; the values were obtained from various satellite observations and reduced to a common selenocentric dynamic frame of reference by robust modeling. Depending on the kind of information content, the data in the base con be conventionally divided into eight structural sections which can be represented as octants. The relation of the digital information to a certain structural section is characterized by the sign of the presented values of coordinates. In total, the DSDC contains 274093 reference objects. The rectangular coordinates $\xi$, $\eta$, $\zeta$ are related to the principal axes of the Moon's inertia and are presented in fractions of the Moon's mean radius of 1738.1 km. The mean calculation accuracy of coordinates in the base for $\xi$, $\eta$, and $\zeta$ does not exceed 0.0001, 0.0001, and 0.0002 of the Moon's mean radius in the absolute value, respectively. The mapping of objects between different databases of lunar objects can be carried out using comparative analysis of closeness of the presented coordinate positions.

The access to the DSDC is implemented in the form of the Heights Comparison software integrated into a simulation package and composed in the MATLAB system. This software gives access to an interactive search engine allowing one to: draw a sample of digital data, create a simulation model of cartographic support for lunar navigational satellite systems (LNSSs), and perform comparative analysis of height data of the model with the selenographic digital map of the Moon's surface by using different user-defined criteria. The software allows one to test the base and draw a sample of observed objects of selenographic rectangular coordinates from lunar objects by input values using spherical selenographic coordinates that characterize the object position on the Moon's surface. One can choose and/or specify the required options: values of the selenographic latitude and longitude, the sample type (a cube or sphere), and the sample radius in fractions of the Moon's mean radius. The final information can be obtained in the form of a database package including all objects falling within the cube or the sphere of the sample. At a later stage, this approach is planned when solving problems of photogrammetric referencing of the spacecraft to electronic maps of the Moon and other navigation problems.

Figures 2 and 3 present the distribution of DSDC objects in the form of 2D and 3D models.

In Fig. 2, selenographic latitudes and longitudes are plotted in degrees on the ordinate and abscissa, respectively. The color diagram characterizes the number of objects.

In Fig. 3, selenographic latitudes and longitudes are plotted in degrees on the $\beta$ and $\lambda$ axes, respectively. The vertical axis and the color diagram characterize the number of objects.

As seen from Figs. 2 and 3, polar regions contain the least number of objects. The distribution over selenographic latitudes is rather uniform both for the visible and for the hidden side of the Moon.

## 5. DETERMINATION OF THE ZERO POINT POSITION IN THE DSDC COORDINATE SYSTEM

To analyze the coordinate system, the position of the coordinate reference center (CRC) of the DSDC system was determined. For this purpose, the position of the reference center of the DSDC coordinate system was determined relative to the Moon's center of mass (MCM) and center of figure (MCF); then, from

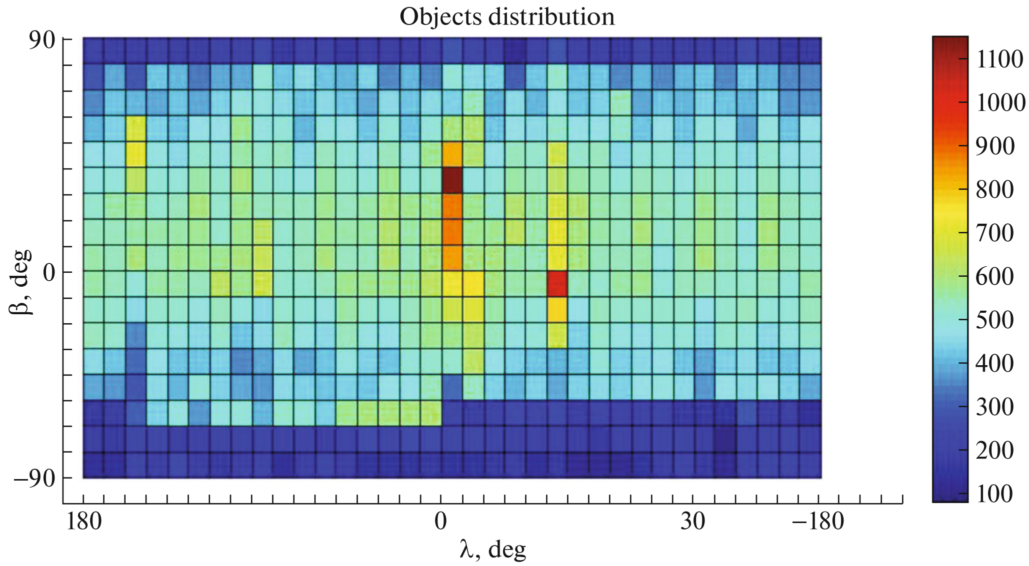


**Fig. 2.** Distribution of DSDC objects in the form of a 2D model.

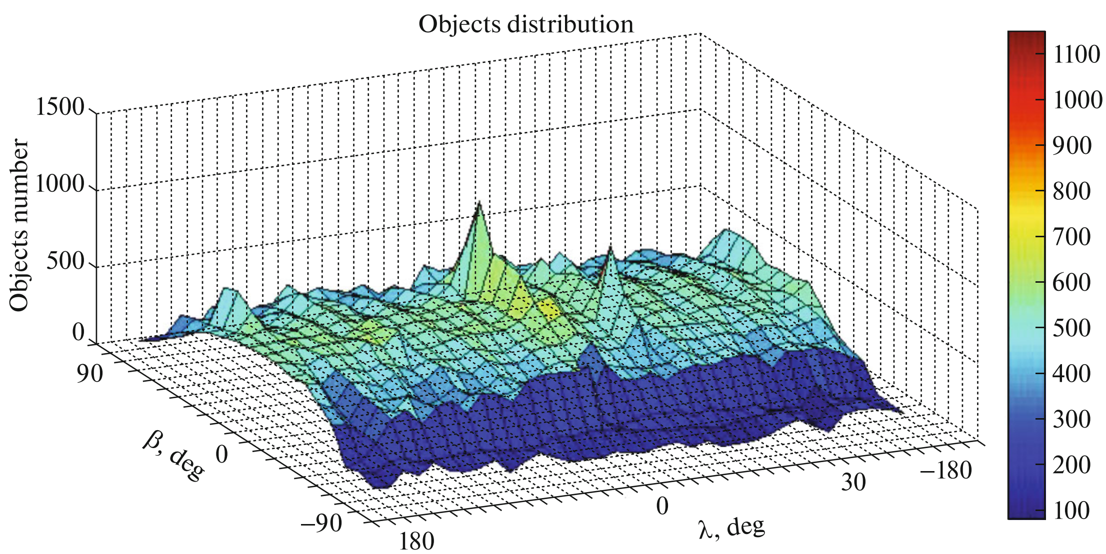


**Fig. 3.** Distributions of CLO objects in the form of a 3D model.

the comparison of this position with values obtained for models of modern missions, the closeness with the MCM is estimated because it is believed that the coordinate systems of the satellite missions are quasi-dynamic, i.e., the CRC coincides with the MCM and the axes do not coincide with the axes of the Moon's inertia.

The CRC position of the DSDC coordinate systems relative to the MCM is modeled in three stages: (i) creation of a multiparameter harmonic model by expansion of DSDC data in spherical functions; (ii) development of the algorithm for estimating harmonic amplitudes; and (iii) determination of the CRC position of the macrofigure model of the DSDC system relative to the MCM and comparison of the obtained results with the satellite observation data by finding positions of the MCM relative to the MCF.

The DSDC data were expanded in spherical functions using the following regression model:

$$h(\lambda, \varphi) = \sum_{n=0}^{N} \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \cos\varphi + \varepsilon, \tag{8}$$

where $\lambda$ and $\varphi$ are the selenocentric longitude and latitude of the crater entering into the CLO system; $C_{nm}$

and $S_{nm}$ are the normalized coefficients; $P_{nm}$ are the normalized associated Legendre functions; and ε are random regression errors. Based on (8), the mathematical model of the physical Moon's surface was created.

The MCF displacement relative to the MCM is determined by the first order harmonics:

$$\Delta\xi = \sqrt{3}\bar{C}_{11}, \quad \Delta\eta = \sqrt{3}\bar{S}_{11}, \quad \Delta\zeta\sqrt{3}\bar{C}_{10}, \tag{9}$$

where $\Delta\zeta$ is the correction to the MCF along the axis coinciding with the direction to the Earth; $\Delta\xi$ is the correction to the MCF along the axis directed perpendicularly to $\zeta$ and lying in the plane of the lunar equator; $\Delta\eta$ is the correction to the MCF along the axis coinciding with the lunar axis of rotation; and $\bar{C}_{11}$, $\bar{S}_{11}$, and $\bar{C}_{10}$ are the first order harmonic amplitudes in the expansion of the regression model.

Based on data of the Clementine and Kaguya space missions and the DSDC, values of normalized first order expansion coefficients of regression model (8) were obtained. As a result, CRC positions from macrofigure models of systems of the Clementine and Kaguya missions and CLO relative to the MCM were determined.

As seen from Table 1, the DSDC shows a good agreement with data of the MCF position relative to the MCM of the Clementine and Kaguya space missions. Therefore, one can conclude that the DSDC coordinate system is related to the dynamic selenocentric coordinate system and can be comprehensively used for carrying out navigation problems both on the Moon's surface and in the near-lunar space.

**Table 1.** Coordinates of the MCF (km) relative to the MCM for three sources of hypsometric information

| | Clementine | Kaguya | DSDC |
|---|---|---|---|
| $\Delta\xi$ | −1.82 | −1.78 | −1.59 |
| $\Delta\eta$ | −0.72 | −0.76 | −0.79 |
| $\Delta\zeta$ | −0.62 | +0.25 | +0.43 |

## 6. HEIGHTS COMPARISON SIMULATION MODEL OF THE DSDC

The method of navigation referencing of near-lunar spacecrafts to the selenocentric dynamic coordinate system with the use of the DSDC was implemented in practice in the form of a simulation package. The simulation model of cartographic support for LNSSs (SMCS) is intended for efficient determination of object coordinates on the Moon's surface and testing of the obtained results. The SMCS is constructed in the form of the Heights Comparison software module and gives access to an interactive search engine allowing one to: create simulation models of cartographic support for LNSSs, draw a sample of digital data, and perform comparative analysis of the model's height data with the selenographic digital map of the Moon's surface by using different user-defined criteria.

The method used in this work for the determination of the selenocentric height of the sought object on the Moon's surface is based on that height data of objects positioned at close distances from each other that are also close, which is peculiar to a lesser extent to objects positioned at considerable distances from each other. Height parameters of sought objects are determined using the weighted altitude parameters (WAP) method. This method acknowledges the identified point to reference objects with known coordinates in the vicinity of the identified object. In the case of a uniform distribution of height samples and stable surface characteristics for different landscape areas, the possibility of exact interpolation of height values of the Moon's surface appears based on known heights of surrounding objects. Different farness of reference objects from the identified point is considered using the approach of assigned weights: larger values of weight are assigned to reference objects that are closer to the identified point. The WAP method is implemented under the assumption that each reference lunar object exerts a local action on the value of the height parameter of the identified object depending on the distance between these objects. Therefore, values assigned to reference objects that are closer to the sought one are higher than for objects positioned at longer distances. Proceeding from these prerequisites, one can write the main expression of the method in the form

$$\hat{H}(O_0) = \sum_{i=1}^{N} P_i H(O_i), \tag{10}$$

where $\hat{H}(O_0)$ is the identified (unknown) value of the height parameter for the object $O_0$ and $P_i$ are specified values of weight for each reference object to be used in the investigations. The weight varies depending on the distance; $H(O_i)$ is the measured (known) value of the height parameter for the object $O_i$; and $N$ is the number of reference objects found near the identified object. These are planned to be used in interpolation procedures.

The weights are calculated according to the following expression:

$$P_i = \frac{D_{i0}^{-p}}{\sum_{i=1}^{N} D_{i0}^{-p}}, \tag{11}$$

where $D_{i0}$ is the distance between the $i$th reference object $O_i$ and identified object $O_0$.

In expression (11), the weight varies depending on the parameter $p$. The power exponent $p$ has an effect

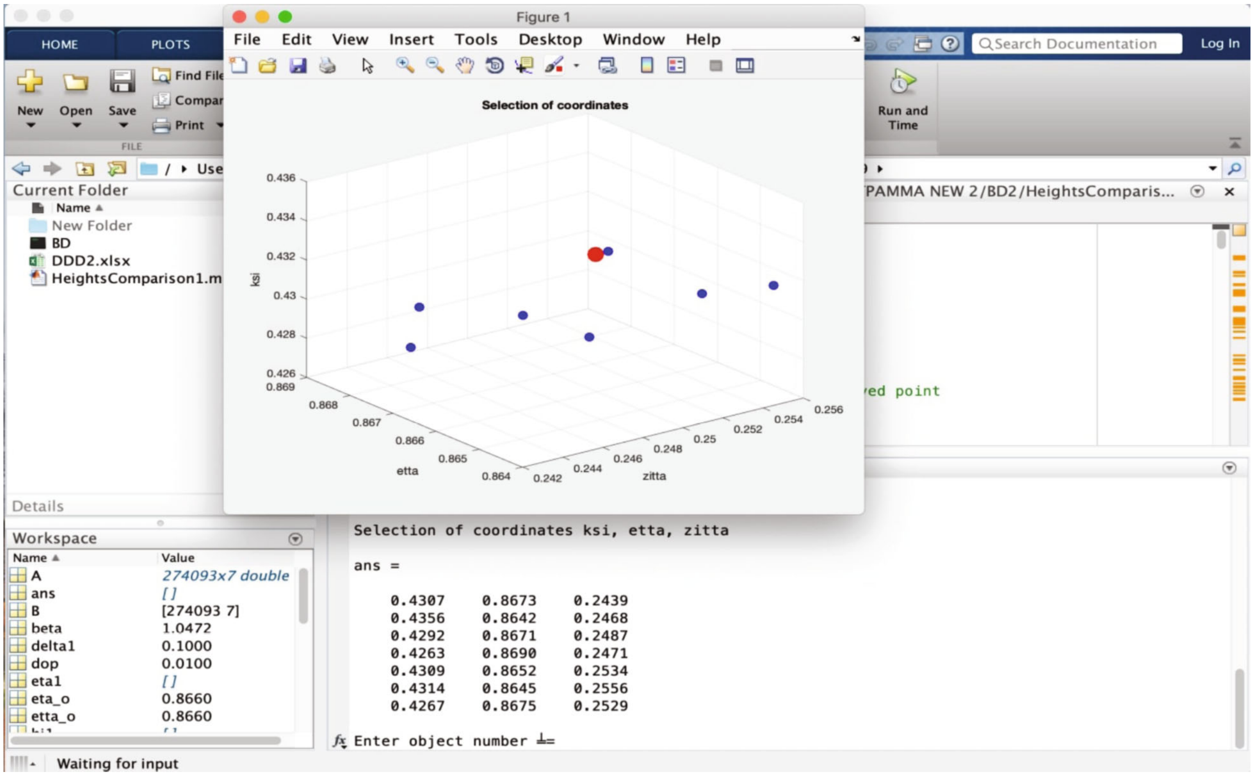


**Fig. 4.** Distribution of objects in the 3D space in the $\xi$, $\eta$, $\zeta$ system.

on which weights will be assigned to reference objects. It follows that the weight assigned to the reference objects will decrease depending on the increase in the distance between the identified and reference objects. Correspondingly, the height parameter of the identified object will also decrease exponentially. Addition of weights for all reference objects involved into the interpolation process yields a sum equal to 1:

$$\sum_{i=1}^{N} \lambda_i = 1. \tag{12}$$

In the course of calculations, seeking for the minimum RMSE, one can determine the optimum value of the parameter $p$.

The SMCS allows one to find selenocentric parameters by values of longitude $\lambda$ and latitude $\beta$ of the object under study in the given region on the Moon's surface based on observations from the spacecraft's board with the use of a digital camera and laser interferometer. Knowing approximate observed coordinates of the sought objects, we obtain the mutual position of the sought object (Fig. 4, red dot) and DSDC objects (Fig. 4, blue dots) in the space. In the presented example, seven DSDC objects were found.

Values of $\xi$, $\eta$, $\zeta$ for the given point and objects entering into the DSDC are determined in automatic mode. In addition, the error curve is formed. This additional information has no effect on further investigations; it yields information about uniformity of the object distribution in the region under study. Then, the height of the sought point $H_n$ is determined in kilometers and the interpolated model height $H_0$ of the region surrounding this point over the Moon's mean radius is calculated in kilometers. At the same time, weight coefficients and distances $d_i$ from the tested point on digital maps to centers of digital map pixels surrounding the point under study are determined. As a result, we obtain the RMSE of height determination by the DSDC and the difference between the height of the sought crater and interpolated height from the Moon's digital map. One should consider that the coordinates ($\lambda$, $\beta$) for the Moon's surface vary within the limits of $\pm 180°$ and $\pm 90°$, respectively. However, to compare heights from the DBC and that obtained from the electronic height map of the Kaguya mission, it is necessary to restrict oneself to limits of $\pm 174°$ and $\pm 84°$ when using the algorithm for obtaining the height $H$ of a given point with coordinates ($\lambda$, $\beta$) by the IDW method. With the aim of testing, values for different octants were obtained. Table 2 analysis shows that values obtained for the Moon's visible side (Octant no. 1) are more accurate than for the hidden side (Octant no. 3).

Table 2 presents the weight **P**, the RMSE **RMS**, the height of the sought point in the reference catalog system $\mathbf{H_N}$, the height for the same object from the

**Table 2.** Comparison of results obtained when determining the height of the point in question for two different octants

| Octant no. 1 | | Octant no. 3 | |
|---|---|---|---|
| **P** | 1 | **P** | 1 |
| **RMS** | 0 | **RMS** | 16.6 |
| $\mathbf{H}_N$ | −2.11 | $\mathbf{H}_N$ | 3.05 |
| **H** | −2.07 | **H** | −0.82 |
| Δ | 0.04 | Δ | −3.87 |
| σ | 0.007 | σ | −2.8 |

electronic height map of the Kaguya mission $\mathbf{H}$, $\Delta = H - H_N$, and the RMSE of height σ. All parameters are given in kilometers. Thus, largest errors are observed for regions on the Moon's hidden side. This again corroborates the fact that the coordinate–time support for the Moon's visible side is more accurate than for the hidden side. This also explains the fact that lunar landing of spacecrafts at present requires correction of their trajectory by an operator in the online mode. As an example, one can present data of the Chinese Chang'e-4 lunar mission. The module's lunar landing of the Chang'e-4 mission became possible only after the establishment of direct communication with the spacecraft via a repeater satellite placed at the Lagrange point.

At a later stage, a simulation model of cartographic support (SMCS) for LNSSs shall be used to solve problems of a spacecraft's photogrammetric referencing to electronic lunar maps and other navigation problems.

## 7. CONCLUSIONS

Upon analysis methods for the creation of LNSSs, present-day selenographic systems are unequal and their data are related to different coordinate systems [10]. Therefore, it is necessary to use methods of space photogrammetry. Furthermore, space observations should be reduced by using both ground absolute observations and exact theory of the Moon's physical libration [11].

The simulation model of cartographic support (SMCS) for LNSSs (SMCS) described in this work allows one to simulate the determination of observed selenographic rectangular coordinates of lunar objects by values of input spherical selenographic coordinates characterizing the object position on the Moon's surface. This system can be used directly in the online mode, both at stationary observation stations and onboard a navigation satellite.

As a result of comparative analysis of the obtained data with hypsometric models constructed based on present-day space missions, the determination accuracy of a chosen point's coordinates on the Moon's surface has been investigated. It has been found that the coordinate–time and navigation support for the Moon's visible side is more accurate than for the hidden side, which corroborates earlier conclusions made in other works [12]. The developed approaches can be used in determining the coordinates of the point in question on the Moon's surface using different navigational goniometric devices, ground positioning tools, laser light beacons [13], high-precision onboard star sensors [14], and other measurement methods.

## FUNDING

This work was supported in part by the Russian Science Foundation, project nos. 20-12-00105 (development of the data analysis method). The work was carried out within the scope of the Russian Government Program of Competitive Growth of Kazan Federal University. The work was supported in part by the scholarship of the President of the Russian Federation to young scientists and postgraduate students SP-3225.2018.3, Russian Foundation for Basic Research, project no. 19-32-90024 Aspirants, and the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS".

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*Translated by A. Nikol'skii*